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Inverted triode amplifiers

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INVERTED TRIODE AMPLIFIERS

by

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I. INTRODUCTION

The specific purpose of this investigation was to study analytically and experimentally the three inverted modes of operation of a triode vacuum tube*:

1. Plate-input grid-output amplifier
2. Plate-input cathode-output amplifier
3. Cathode-input grid-output amplifier

The three inverted modes of operation are analogs, respectively, of plate-loaded amplifiers, cathode-follower amplifiers, and grounded-grid amplifiers.

The investigation included studies of the static characteristic curves of typical triodes and of the inverse mutual conductance, inverse amplification factor, and the dynamic grid resistance in the various regions of inverted operation. Analytical expressions which were derived include the general and specialized expressions for gain, input admittance, and output admittance for the three inverted modes of operation. Typical numerical values for gain, input admittance, and output admittance are calculated. A study was made of the

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*In inverted operation the roles of the grid and plate, in respect to normal operation, are interchanged. The plate is operated with a negative bias voltage and the grid is operated at a positive voltage. Current flows in the grid circuit and not in the plate circuit for most types of inverted operation.

equipotential lines in an inverted triode. Experimental values of gain and distortion for inverted amplifiers under typical operating conditions are shown.

Several triodes with different electrical characteristics and different mechanical construction were studied. The general behavior of all types studied appeared to be similar. For this reason, only one type, the 6J5, was studied in detail.

II. REVIEW OF LITERATURE

The inverted characteristics of triodes and the operation of triodes in inverted circuits have received very little critical study. A few applications have been made of the plate-input grid-output circuit in high voltage vacuum-tube voltmeters. No references were found describing applications of either plate-input cathode-output or cathode-input grid-output amplifiers. Some study has been made of the characteristic curves of triodes in the region of inverted operation.

Van der Bijl[1] is apparently the first worker in the field to describe a circuit for an inverted triode. He shows a diagram for a high voltage vacuum-tube voltmeter using an inverted triode and credits the circuit to Dr. E. H. Stoekle. His description of the circuit is brief and does not include any critical analysis.

Terman[2] gives a rather complete description of the operation of a vacuum-tube voltmeter using an inverted triode. He shows the static curves of triodes for the inverted region

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[1]H. J. Van der Bijl. The thermionic tube and its applications. New York, McGraw-Hill Book Co., Inc. 1920. pp. 369-371.

[2]Frederick E. Terman. The inverted vacuum tube, a voltage reducing power amplifier. Proc. I.R.E. 16: 447-461. 1928.

of operation, but his curves are regular and do not show any peculiarities. He derives an expression for the low-frequency gain, and he includes a qualitative discussion of the factors which affect the input impedance. He also writes a brief description of an oscillator circuit using an inverted triode.

Schneeberger[1] describes a vacuum-tube voltmeter which employs an inverted tetrode. In his circuit the screen grid of the tetrode is used for stabilization purposes. Several other workers in the field, using Van der Bijl's circuit or slight modifications of his circuit, do not go into any operational details. The articles describing the work of these experimenters are included in the references.

Van der Pol[2] made an extensive study of the static characteristics of triodes for various combinations of positive and negative plate and grid voltages. He notes that there are peculiarities in the curves for positive grid voltages and low negative plate voltages and he attributes these peculiarities to space charge effects. Most of Van der Pol's discussion is, however, concerned with the characteristics of triodes in the regions used for conventional amplifier

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[1]R. J. Schneeberger. An inverted tetrode voltmeter for high negative voltages. Rev. Sci. Inst. 19: 40-42. 1948.

[2]Balth. Van der Pol, Jr. Über Elektronenbahnen in Trioden. Zeits. f. Hochfrequenztechnik. 25: 121-131. 1925.

operation.

Greve[1], in an abstract of his dissertation at Jena, extends Van der Pol's work and pays special attention to the peculiarities of the static characteristics of triodes in the inverted region of operation. His explanations of the reasons for the peculiarities are mostly qualitative and are little different from those advanced by Van der Pol. He states that the spacing of the grid wires in a triode has little effect on the general shape of the characteristic curves in the inverted region.

Chaffee[2] gives an extended discussion of the characteristic curves of triodes. Much of his discussion is essentially an English version of Van der Pol's article, and he uses Van der Pol's curves with English captions for most of his illustrations.

Ryder[3] is apparently the only worker in the field to suggest the possibility of using an inverted triode in other than a plate-input grid-output amplifier. He shows the circuits of plate-input cathode-output and cathode-input

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[1] Ferdinand Greve. Untersuchungen über den Durchgriff von Empfängerröhren. Zeits. f. Hochfrequenztechnik. 38: 234-237. 1931.

[2] E. Leon Chaffee. Theory of thermionic vacuum tubes. New York, McGraw-Hill Book Co., Inc. 1933. Chpt. VII.

[3] John D. Ryder. Electronic fundamentals and applications. New York, Prentice-Hall, Inc. 1950. pp. 242-244.

grid-output amplifiers but makes no effort to analyze the behavior of these circuits. His brief statements, however, led to the present investigation.

III. INVESTIGATION

A. General Scope of Investigation

The investigation may be divided into two main parts. The first part is concerned with the mathematical treatment of inverted amplifiers. The second part is devoted to an experimental study of the characteristics of inverted triodes and their behavior in various circuits.

Equations for gain, input admittance, and output admittance for the three types of inverted amplifiers are derived. The derivations are made on a small signal basis, i.e., signal voltages are assumed small enough that operation is over essentially linear portions of the characteristic curves of the tubes. The equations are derived for the general case and include the effects of interelectrode capacitances and any type of static load impedance. Specialized forms of the equations, applicable to certain conditions of operation, are also derived. Comparisons with analogous equations for conventional amplifiers are made. A-c equivalent circuits for inverted triodes are derived. The equipotential lines in a plane-electrode triode are computed and plotted for the condition of positive grid voltage and negative plate voltage.

The second part of the investigation is concerned with the various characteristics of inverted triodes and associated circuits. A study is made of the static characteristic curves of triodes with positive grid voltages and negative plate voltages. A detailed study is made of the inverted region for low negative plate voltages because of peculiarities in the characteristic curves which were found in this region. Families of curves are shown for the inverse amplification factor, inverse mutual conductance, and dynamic grid resistance as functions of grid current. Negative plate voltage is taken as the parameter for these curves. Measured values of gain and harmonic distortion of the three types of inverted amplifiers under various operating conditions are shown.

B. Mathematical Analysis

1. Equivalent circuits for an inverted triode.

The mathematical analysis of vacuum-tube circuits is greatly simplified if the region of operation on the characteristic curves of the tube is restricted so that linear equations are sufficiently accurate to describe the behavior. This restriction implies a judicious choice of the size of the direct voltages, which fix the quiescent point of operation, and the size of the alternating signal voltages and circuit impedances, which determine the dynamic swings of voltages and currents. The signal voltages must ordinarily be rather small so that operation will be linear, and this method of analysis is termed "small-signal" analysis.

The usual approach to the small-signal analysis of a vacuum-tube circuit is to first derive an a-c equivalent circuit for the tube. This equivalent circuit replaces the tube in subsequent steps of the analysis. Other circuit elements are connected to the equivalent circuit of the tube at appropriate points. The parameters of the equivalent circuit are determined primarily by the choice of the quiescent operating point. Direct voltages and currents do not appear in the equivalent circuit except through their effect on the equivalent circuit parameters. Thus, the analysis problem is effectively resolved to that of solving an a-c circuit problem. This is the approach taken in deriving equations for gain, input admittance, and output admittance for the three inverted amplifiers.

The grid current in a triode operated with its plate negative and its grid positive is a function of both the plate and grid voltages. In general, the grid current decreases as the plate voltage is made more negative, and increases as the grid voltage is made more positive. Thus, the behavior of an inverted triode is the same as that of a triode in normal operation if the roles of the grid and plate are interchanged and a positive voltage is applied to the plate and a negative voltage is applied to the grid.

Before proceeding to a formal analysis of the operation of inverted triodes, it will be useful to state the letter symbols[1] which are used in the following discussion.

e_c	Instantaneous total grid voltage
e_b	Instantaneous total plate voltage
i_c	Instantaneous total grid current
i_b	Instantaneous total plate current
E_c	Average or quiescent value of grid voltage
E_b	Average or quiescent value of plate voltage
I_c	Average or quiescent value of grid current
I_b	Average or quiescent value of plate current
e_g	Instantaneous value of varying component of grid voltage
e_p	Instantaneous value of varying component of plate voltage
i_g	Instantaneous value of varying component of grid current
i_p	Instantaneous value of varying component of plate current
E_g	Effective value of varying component of grid voltage
E_p	Effective value of varying component of plate voltage
I_g	Effective value of varying component of grid current
I_p	Effective value of varying component of plate current
E_s	Effective value of signal voltage
g_g	Dynamic grid conductance
g_p	Dynamic plate conductance
r_g	Dynamic grid resistance
r_p	Dynamic plate resistance
g_m	Grid-plate transconductance (mutual conductance)
g_n	Plate-grid transconductance (inverse mutual conductance)
μ	Amplification factor
μ_n	Inverse amplification factor
C_{gp}	Grid-plate capacitance
C_{pk}	Plate-cathode capacitance
C_{gk}	Grid-cathode capacitance

[1] These symbols are adapted from Standards on electronics. Definitions of terms, symbols. New York, The Institute of Radio Engineers. 1938 (reprinted, 1943). Consistent symbols are adopted where no standard letter symbol is specified.

The functional expression for grid current is given by

$$i_c = f(e_c, e_b) \; . \tag{1}$$

The total differential of this expression is

$$di_c = \frac{\partial i_c}{\partial e_c}\, de_c + \frac{\partial i_c}{\partial e_b}\, de_b \; . \tag{2}$$

The dynamic grid conductance, g_g, is defined as $\partial i_c/\partial e_c$ and the inverse mutual conductance, g_n, is defined as $\partial i_c/\partial e_b$. Then

$$di_c = g_g de_c + g_n de_b \; . \tag{3}$$

Now let $di_c = 0$. This is equivalent to investigating the behavior under the condition of constant total grid current. The result is

$$0 = g_g de_c + g_n de_b$$

or

$$\frac{g_n}{g_g} = - \left. \frac{de_c}{de_b} \right|_{di_c = 0} = \mu_n \; . \tag{4}$$

The series and parallel a-c equivalent circuits may be derived in a manner analogous to that employed for triodes operated in a normal fashion.[1] The resulting a-c equivalent circuits are shown in Fig. 1. The various interelectrode capacitances are not shown, but can be added at the appropriate places when needed. The important restriction which must be observed when using these equivalent circuits is that the

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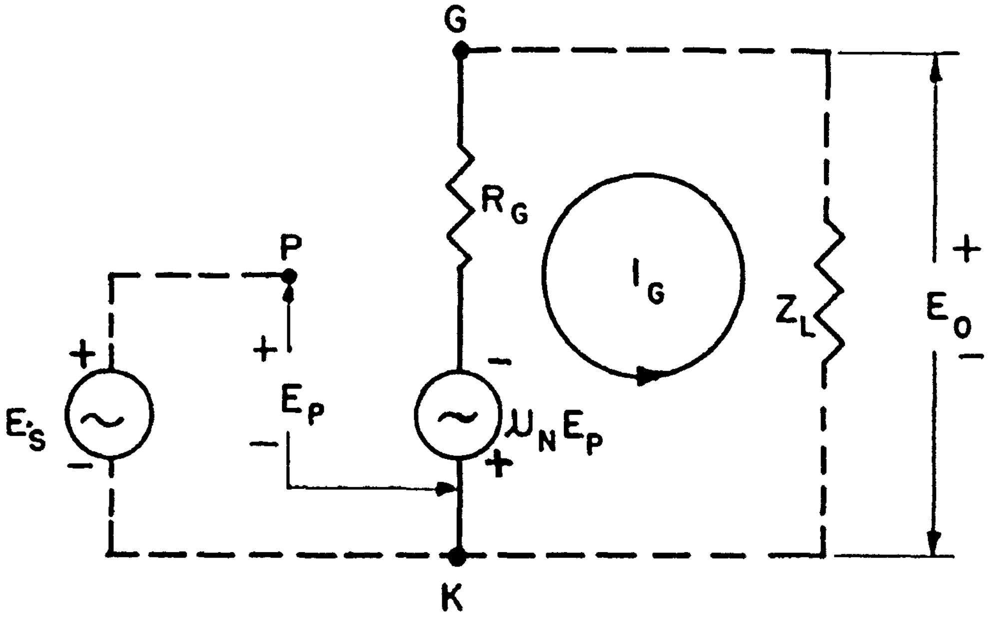
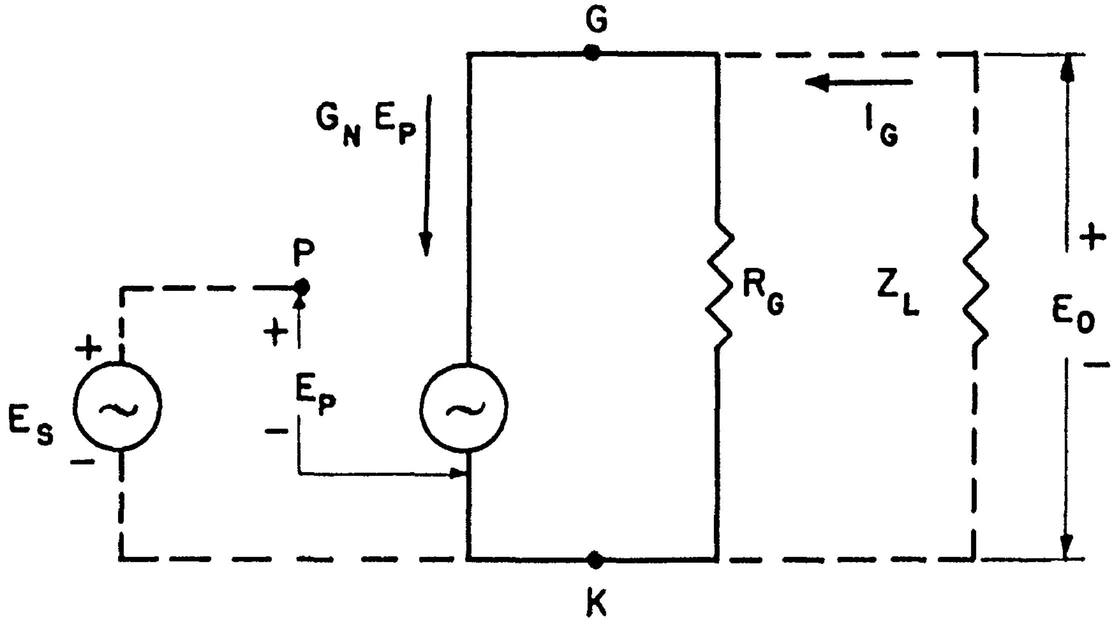
[1]Ryder, op. cit., pp. 198-204.



(a) Series-equivalent



(b) Parallel-equivalent

Fig. 1. Series and Parallel A-c Equivalent Circuits for an Inverted Triode.

quiescent point and the signal voltage must be selected so that the path of operation does not depart from a linear portion of the dynamic characteristics of the tube. The equivalent circuits are labeled with symbols for sinusoidal operation. Instantaneous values of the varying components of grid and plate voltages and currents may be employed if the restrictions upon the region of operation are observed.

The parts of the circuits in Fig. 1 which are drawn with solid lines are those belonging to the equivalent circuit of the tube. The parts connected by dashed lines indicate the method of connection in a complete circuit. The complete circuit shown is that of a plate-input grid-output amplifier. The directions of current and of voltage rises are chosen so that they are consistent and will most easily yield the proper phase relationships in expressions for gain. Other choices may be made if proper attention is paid to algebraic signs.

Even though the a-c equivalent circuits are strictly applicable only when operation is over a limited region of a tube's characteristic curves, these equivalent circuits may also be used to obtain an estimate of the behavior of a tube in a circuit in which the restriction is not observed. When the equivalent circuits are used in this manner, too much importance must not be attached to the numerical results obtained. However, if reasonable discretion is used in interpreting results, much useful information can be gained without the complications arising from an attempt to solve non-linear equations.

2. <u>Gain, input admittance, and output admittance.</u>

Three characteristics of amplifier circuits which may be obtained by the use of a-c equivalent circuits are of particular importance. They are gain, input admittance (or impedance), and output admittance (or impedance). Alternate methods are available for obtaining expressions for these three characteristics. One method is to utilize the a-c equivalent circuit for the inverted triode. The circuit which is being studied is connected to the equivalent circuit for the tube and the analysis carried forward by the use of conventional a-c circuit theory. The second method is to derive expressions for these characteristics of the three amplifiers employing triodes in a conventional manner, and then to convert the resulting expressions by use of the method of analogs. The last method, while it seems to be circuitous, offers important advantages. Many of the expressions have been derived by previous workers in the field, are easily available in the literature, and can be used for a check on the accuracy of derived expressions. A second advantage is that offered by familiarity with conventional amplifier circuits. The second method was used in deriving the expressions presented herein.

General expressions for gain, input admittance, and output admittance of conventional amplifier circuits were not found in the literature in many cases. Often simplifying assumptions were made near the beginning of a derivation thus making the resulting expressions special cases. However, the special cases were useful for partial checks. Ryder[1] and Arguimbau[2] were found to be particularly useful in checking derived expressions.

The actual and the series a-c equivalent circuits for the three types of inverted amplifiers are shown in Figs. 2, 3, and 4. The derived expressions for gain, input admittance, and output admittance are shown in Tables 1, 2, and 3, respectively. The expressions for gain, input admittance, and output admittance for conventional amplifiers are shown in Tables 4, 5, and 6.

In all cases the general expressions for gain, input admittance, and output admittance are calculated. That is, both the interelectrode capacitances and a general static load impedance are taken into consideration. Specialized expressions are derived in addition to the general expressions. The most significant specialized expressions are those in

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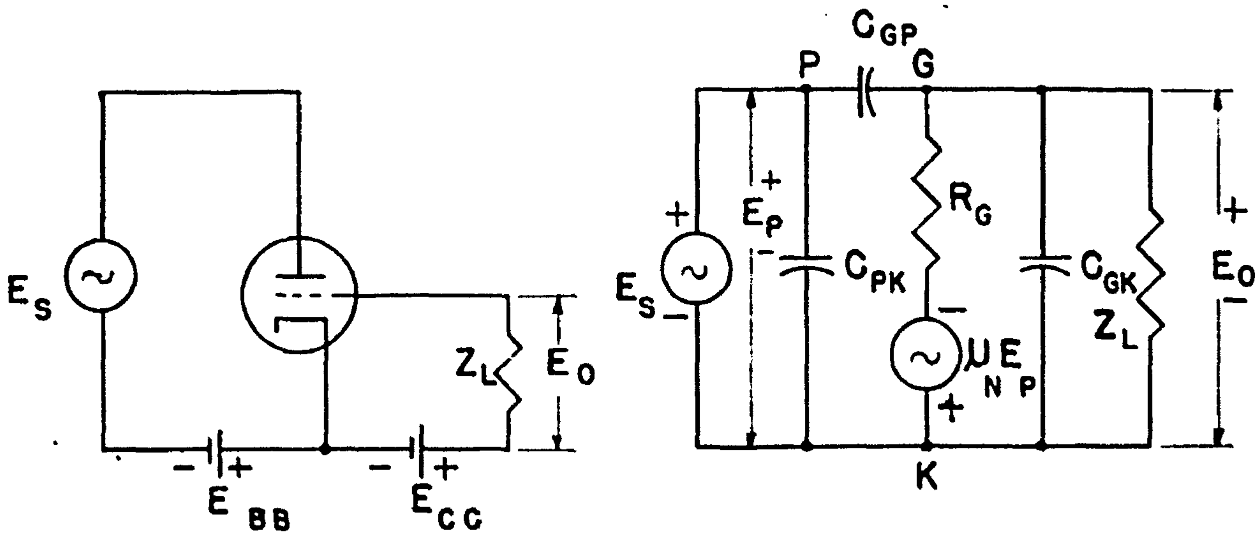
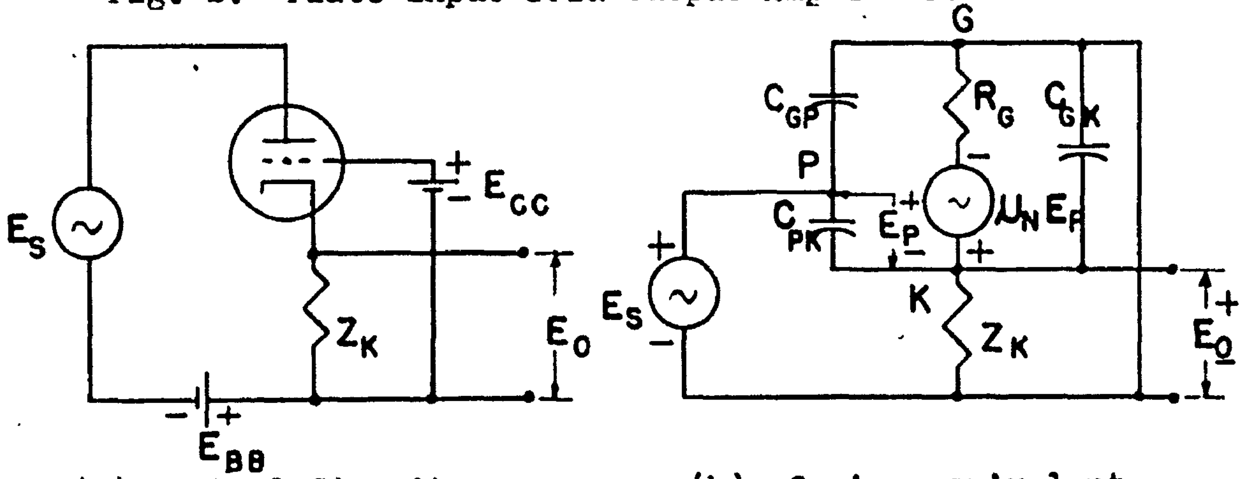
[1]Ryder, op. cit.

[2]Lawrence B. Arguimbau. Vacuum-tube circuits. New York, John Wiley & Sons, Inc., 1948.



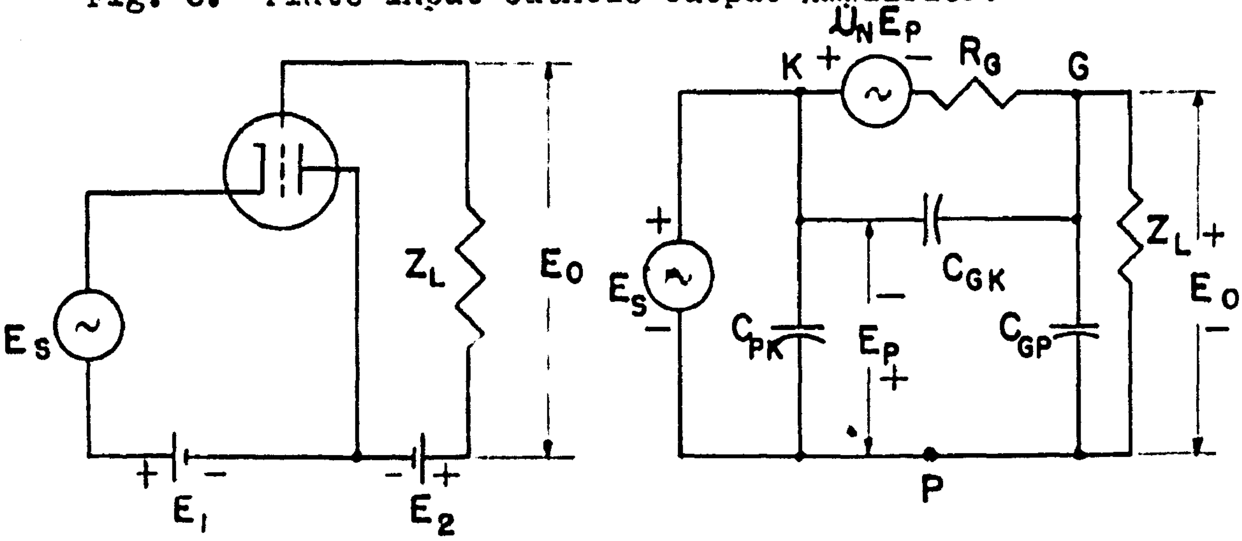
(a) Actual Circuit (b) Series-equivalent

Fig. 2. Plate-input Grid-output Amplifier.



(a) Actual Circuit (b) Series-equivalent

Fig. 3. Plate-input Cathode-output Amplifier.



(a) Actual Circuit (b) Series-equivalent

Fig. 4. Cathode-input Grid-output Amplifier.

Table 1

Voltage Gain of Inverted Amplifiers

Plate-input Grid-output	Plate-input Cathode-output	Cathode-input Grid-output
<u>General</u>	<u>General</u>	<u>General</u>
$A = -\dfrac{\left(\mu_n - \frac{r_g}{Z_2}\right) Z_L'}{r_g\left(1 + \frac{Z_L'}{Z_2}\right) + Z_L'}$	$A = \dfrac{g_n Z_2 + 1}{g_n Z_2 + 1 + \frac{Z_2(Z_K' + r_g)}{Z_K' r_g}}$	$A = \dfrac{[r_g + Z_3(\mu_n + 1)] Z_L'}{Z_3(r_g + Z_L') + r_g Z_L'}$
<u>$C_{gp} = C_{gk} = C_{pk} = 0$</u>	<u>$C_{gp} = C_{gk} = C_{pk} = 0$</u>	<u>$C_{gp} = C_{gk} = C_{pk} = 0$</u>
$A = -\dfrac{\mu_n Z_L}{r_g \quad Z_L}$	$A = \dfrac{\mu_n Z_K}{r_g + (\mu_n + 1) Z_K}$	$A = \dfrac{(\mu_n + 1) Z_L}{r_g + Z_L}$
	<u>$C_{gp} = C_{gk} = C_{pk} = 0,\ 1 \gg \mu_n$</u>	<u>$C_{gp} = C_{gk} = C_{pk} = 0,\ 1 \gg \mu_n$</u>
	$A = \dfrac{\mu_n Z_K}{r_g + Z_K}$	$A = \dfrac{Z_L}{r_g + Z_L}$
$Z_L' = 1/Y_L'$	$Z_K' = 1/Y_K'$	$Z_L' = 1/Y_L'$
$Y_L' = Y_L + j\omega C_{gk}$	$Y_K' = Y_K + j\omega C_{gk}$	$Y_L' = Y_L + j\omega C_{gp}$
$Z_2 = -j/\omega C_{gp}$	$Z_2 = -j/\omega C_{pk}$	$Z_3 = -j/\omega C_{gk}$

Table 2

Input Admittance of Inverted Amplifiers

Plate-input Grid-output	Plate-input Cathode-output	Cathode-input Grid-output
<u>**General**</u> $Y_{in} = j\omega[C_{pk} + C_{gp}(1 - A)]$	<u>**General**</u> $Y_{in} = j\omega[C_{gp} + C_{pk}(1 - A)]$	<u>**General**</u> $Y_{in} = j\omega C_{pk} + \frac{A}{Z_L'}$ $= j\omega C_{pk} + A(Y_L + j\omega C_{gp})$
<u>$\lvert A\rvert \ll 1$</u> $Y_{in} = j\omega(C_{pk} + C_{gp})$	<u>$\lvert A\rvert \ll 1$</u> $Y_{in} = j\omega(C_{gp} + C_{pk})$	

$Z_L' = 1/Y_L'$

$Y_L' = Y_L + j\omega C_{gp}$

Table 3

Output Admittance of Inverted Amplifiers

Plate-input Grid-output	Plate-input Cathode-output
General	General
$Y_o = j\omega C_{gk} + \frac{(Z_2 + r_g)(Z_1 + Z_S) + Z_1 Z_S(\mu_n + 1)}{r_g(Z_1 Z_2 + Z_1 Z_S + Z_2 Z_S)}$	$Y_o = Y_3 + \frac{(g_n + Y_2)(Y_1 + Y_S)}{Y_1 + Y_2 + Y_S}$
$Z_S = 0$	$Z_S = 0$ $(Y_S = \infty)$
$Y_o = g_g + j\omega(C_{gp} + C_{gk})$	$Y_o = g_n + Y_2 + Y_3$
$Z_S = \infty$	$Z_S = \infty$ $(Y_S = 0)$
$Y_o = \left(g_g + g_n \frac{C_{gp}}{C_{gp} + C_{pk}}\right) + j\omega\left(C_{gk} + \frac{C_{gp} C_{pk}}{C_{gp} + C_{pk}}\right)$	$Y_o = Y_3 + \frac{Y_1(Y_2 + g_n)}{Y_1 + Y_2}$
$C_{gp} = C_{gk} = C_{pk} = 0$	$C_{gp} = C_{gk} = C_{pk} = 0$
$Y_o = g_g = \frac{1}{r_g}$	$Y_o = Y_K + g_g + g_n$

Z_S = Impedance of signal source

$Z_1 = -j/\omega C_{pk}$

$Z_2 = -j/\omega C_{gp}$

Z_S = Impedance of signal source

$Y_S = 1/Z_S$ $\quad Y_K = 1/Z_K$

$Y_1 = j\omega C_{gp}$

$Y_2 = j\omega C_{pk}$

$Y_3 = g_g + Y_K + j\omega C_{gk}$

fiers

-input Cathode-output	Cathode-input Grid-output
<u>al</u>	<u>General</u>
$Y_3 + \frac{(g_n+Y_2)(Y_1+Y_S)}{Y_1+Y_2+Y_S}$	$Y_o = j\omega C_{gp} + \frac{(r_g+Z_3)(Z_1+Z_S)}{r_g Z_3(Z_1+Z_S)+Z_1 Z_S(r_g+Z_3+\mu_n Z_3)}$
<u>) $(Y_S=\infty)$</u>	<u>$Z_S=0$</u>
$g_n + Y_2 + Y_3$	$Y_o = g_g + j\omega(C_{gp} + C_{gk})$
<u>o $(Y_S=0)$</u>	<u>$Z_S=\infty$</u>
$Y_3 + \frac{Y_1(Y_2+g_n)}{Y_1+Y_2}$	$Y_o = j\omega\left[C_{gp}+C_{pk}\left(\frac{g_g+j\omega C_{gk}}{g_g(\mu_n+1)+j\omega(C_{pk}+C_{gk})}\right)\right]$
<u>$_{gk}=C_{pk}=0$</u>	<u>$C_{gp}=C_{gk}=C_{pk}=0$</u>
$Y_K + g_g + g_n$	$Y_o = \frac{1}{r_g+Z_S(\mu_n+1)}$

Impedance of signal source

$1/Z_S \quad Y_K = 1/Z_K$

$j\omega C_{gp}$

$j\omega C_{pk}$

$g_g + Y_K + j\omega C_{gk}$

Z_S = Impedance of signal source

$Z_1 = -j/\omega C_{pk}$

$Z_3 = -j/\omega C_{gk}$

Table 4

Voltage Gain of Conventional Amplifiers

Plate-loaded	Cathode-follower	Grounded-grid
General	**General**	**General**
$A = -\dfrac{(\mu Z_2 - r_p)Z_L'}{r_p(Z_2 + Z_L') + Z_2 Z_L'}$	$A = \dfrac{g_m Z_2 + 1}{g_m Z_2 + 1 + \dfrac{Z_2(Z_K' + r_p)}{Z_K' r_p}}$	$A = \dfrac{[r_p + Z_3(\mu + 1)]\, Z_L'}{Z_3(r_p + Z_L') + r_p Z_L'}$
$\underline{C_{gp} = C_{gk} = C_{pk} = 0}$	$\underline{C_{gp} = C_{gk} = C_{pk} = 0}$	$\underline{C_{gp} = C_{gk} = C_{pk} = 0}$
$A = -\dfrac{\mu Z_L}{r_p + Z_L}$	$A = \dfrac{\mu Z_K}{r_p + (\mu + 1)Z_K}$	$A = \dfrac{(\mu + 1)Z_L}{r_p + Z_L}$
$Z_L' = 1/Y_L'$	$Z_K' = 1/Y_K'$	$Z_L' = 1/Y_L'$
$Y_L' = Y_L + j\omega C_{pk}$	$Y_K' = Y_K + j\omega C_{pk}$	$Y_L' = Y_L + j\omega C_{gp}$
$Z_2 = -j/\omega C_{gp}$	$Z_2 = -j/\omega C_{gk}$	$Z_3 = -j/\omega C_{pk}$

Table 5

Input Admittance of Conventional Amplifiers

Plate-loaded	Cathode-follower	Grounded-grid
General $Y_{in} = j\omega\left[C_{gk} + C_{gp}(1 - A)\right]$	General $Y_{in} = j\omega\left[C_{gp} + C_{gk}(1 - A)\right]$ A = 1 $Y_{in} = j\omega C_{gp}$	General $Y_{in} = j\omega C_{gk} + A/Z_L'$ $= j\omega C_{gk} + A(Y_L + j\omega C_{gp})$

$Z_L' = 1/Y_L'$

$Y_L' = Y_L + j\omega C_{gp}$

Table 6

Output Admittance of Conventional Amplifiers

Plate-loaded	Cathode-follower
<u>General</u>	<u>General</u>
$Y_o = j\omega C_{pk} + \dfrac{(Z_2+r_p)(Z_1+Z_S)+Z_1Z_S(\mu+1)}{r_p(Z_1Z_2+Z_1Z_S+Z_2Z_S)}$	$Y_o = Y_3 + \dfrac{(g_m+Y_2)(Y_1+Y_S)}{Y_1+Y_2+Y_S}$
<u>$Z_S=0$</u>	<u>$Z_S=0$</u>
$Y_o = g_p + j\omega(C_{gp}+C_{pk})$	$Y_o = g_m+Y_2+Y_3$
<u>$Z_S= \infty$</u>	<u>$Z_S= \infty$</u>
$Y_o = \left(g_p + g_m\dfrac{C_{gp}}{C_{gp}+C_{gk}}\right) + j\omega\left(C_{pk}+\dfrac{C_{gp}C_{gk}}{C_{gp}+C_{gk}}\right)$	$Y_o = Y_3 + \dfrac{Y_1(Y_2+g_m)}{Y_1+Y_2}$
<u>$C_{gp}=C_{gk}=C_{pk}=0$</u>	<u>$C_{gp}=C_{gk}=C_{pk}=0$</u>
$Y_o = g_p = 1/r_p$	$Y_o = Y_K+g_p+g_m$

Z_S = Impedance of signal source

$Z_1 = -j/\omega C_{gk}$

$Z_2 = -j/\omega C_{gp}$

Z_S = Impedance of signal source

$Y_S = 1/Z_S \quad Y_K = 1/Z_K$

$Y_1 = j\omega C_{gp} \quad Y_2 = j\omega C_{gk}$

$Y_3 = g_p + Y_K + j\omega C_{pk}$

ers

e-follower	Grounded-grid
	General
$\frac{g_m+Y_2)(Y_1+Y_S)}{Y_1+Y_2+Y_S}$	$Y_o=j\omega C_{gp}+\frac{(r_p+Z_3)(Z_1+Z_S)}{r_pZ_3(Z_1+Z_S)+Z_1Z_S(r_p+Z_3)+\mu Z_1Z_3Z_S}$
	$Z_S=0$
$+Y_3$	$Y_o=g_p+j\omega(C_{gp}+C_{pk})$
	$Z_S=\infty$
$\frac{1(Y_2+g_m)}{Y_1+Y_2}$	$Y_o=j\omega\left[C_{gp}+C_{gk}\frac{g_p+j\omega C_{pk}}{g_p(\mu+1)+j\omega(C_{gk}+C_{pk})}\right]$
$k=0$	**$C_{gp}=C_{gk}=C_{pk}=0$**
$+g_m$	$Y_o=\frac{1}{r_p+Z_S(\mu+1)}$
ance of signal e	Z_S = Impedance of signal source
$Y_K = 1/Z_K$	$Z_1 = -j/\omega C_{gk}$
$Y_2 = j\omega C_{gk}$	$Z_3 = -j/\omega C_{pk}$
$K + j\omega C_{pk}$	

which the effect of interelectrode capacitances are neglected. These yield equations which are valid for low-frequency operation. Other simplified expressions are obtained by assuming the magnitude of the gain to be considerably smaller than unity. This is a reasonable assumption except in the case of the cathode-input grid-output amplifier.

In the case of output admittance, the effect of the series impedance of the signal source is included, and specialized expressions are obtained for limiting values of source impedance.

The plate-input grid-output amplifier gives a phase shift, due to the tube, of 180° from input to output. There may be additional phase shift resulting from reactances in the circuit. The magnitude of the gain has a maximum value, under ordinary conditions of operation, equal to the inverse amplification factor of the tube. Since the inverse amplification factor is approximately the reciprocal of the normal amplification factor, the gain of this amplifier is considerably less than unity. The input admittance is essentially that due to the grid-plate and plate-cathode capacitances in parallel, plus any distributed and wiring capacitance. The Miller effect, which is so significant in determining the input admittance of conventional plate-loaded amplifiers, plays a very minor role in the plate-input grid-output amplifier. The conductive component of the input admittance is determined primarily by the surface leakage between the plate and cathode terminals. In spite of the low gain, in a voltage sense, there is high power amplification at frequencies up to at least 100 kc.

At low frequencies, the output admittance is equal to the dynamic grid conductance. At higher frequencies this value is modified by the effects of the interelectrode capacitances. In general, the output admittance increases with frequency, but not in direct proportion.

The plate-input cathode-output amplifier, like the conventional cathode-follower, has phase shift between the input and output terminals which results from the reactances of the circuit alone. The tube does not produce the 180° phase shift that it does in the plate-input grid-output amplifier. The magnitude of the gain has a maximum value equal approximately to the inverse amplification factor of the tube. Except for phase shift, the gain of the plate-input cathode-output amplifier and the plate-input grid-output amplifier, with equal load impedances, is the same at frequencies low enough that the interelectrode capacitances may be neglected.

The input admittance of a plate-input cathode-output amplifier is nearly the same as that of a plate-input grid-output amplifier. Under the usually valid assumption that the magnitude of the voltage gain is much less than unity, the values of input admittance of the two amplifiers are identical. The conductive component of the input admittance is determined primarily by the surface leakage between the plate and grid terminals. The power amplification is high.

At low frequencies, the output admittance is equal to the sum of the cathode (load) admittance, the dynamic grid conductance, and the inverse mutual conductance. Thus, the output admittance is somewhat higher than for a plate-input grid-output amplifier. The output admittance increases with frequency, but not in direct proportion. It decreases somewhat with an increase in the impedance of the signal source.

The voltage gain of the cathode-input grid-output amplifier has a maximum value slightly greater than unity. There is zero phase shift from input to output resulting from the action of the tube. The only phase shift arises from reactances of the associated circuit. Since the gain is approximately unity under ordinary conditions of operation, the input admittance is approximately the sum of the susceptance of the plate-cathode and grid-plate capacitances and the admittance of the load. The load would usually have a rather large admittance, and the input admittance will generally be high. This is equivalent to a low input impedance.

At low frequencies, the output impedance is the sum of the dynamic grid resistance and a term which is approximately equal to the impedance of the signal source. The output admittance is the reciprocal of the output impedance. In general, the output admittance increases with frequency and decreases as the source impedance is increased.

3. Equipotential lines.

An estimate of the trajectory of electrons in a tube can be obtained from plots of the equipotential lines in a tube without space charge. Such plots are relatively easy to obtain for tubes with simple geometries since the equations of equipotential and flow lines of the electric field must satisfy Laplace's equation.

A plane-electrode tube was selected for the study of the equipotential lines. This type of tube was chosen in preference to a cylindrical triode because of its simple geometry. However, the electric fields in a plane-electrode and a cylindrical tube are related through complex variable transformations.[1]

For purposes of the study a tube having a geometrical amplification factor of 20 was chosen.[2] The ratio of grid-wire diameter to spacing between grid wires was 0.08. The ratio of grid-to-plate spacing to spacing between grid wires was 4.4. The grid-to-cathode spacing was equal to the spac-

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[1]Karl R. Spangenberg. Vacuum tubes. New York, McGraw-Hill Book Co., Inc. 1948. pp. 127, 135.

[2]Ibid., pp. 126, 149.

ing between grid wires. The potential was calculated at eleven points on each of five lines extending from the cathode to the plate. Two of these lines passed through adjacent grid wires, two more lines were located one-fourth of the distance from one grid to another, and the other line passed half way between the grid wires. These calculations yielded the potential at 55 points in the region between two adjacent grid wires and extending from the cathode to the plate. These points of known potential gave sufficient information from which equipotential lines were plotted for several plate and grid voltages. These plots are seen in Figs. 5 and 6. In all cases the grid was at a positive potential of 5 volts. The plate potential was varied from cathode voltage to -120 volts. Cathode potential was assumed to be zero and taken as the point of voltage reference. The plot for -120 volts of plate potential shows the equipotentials in the tube when it is in a cut-off condition. The plate voltage for cut-off was -100 volts. Other plots are for negative plate voltages less than the cut-off value.

C. Experimental Data

1. Grid dissipation.

One of the first problems which must be considered in

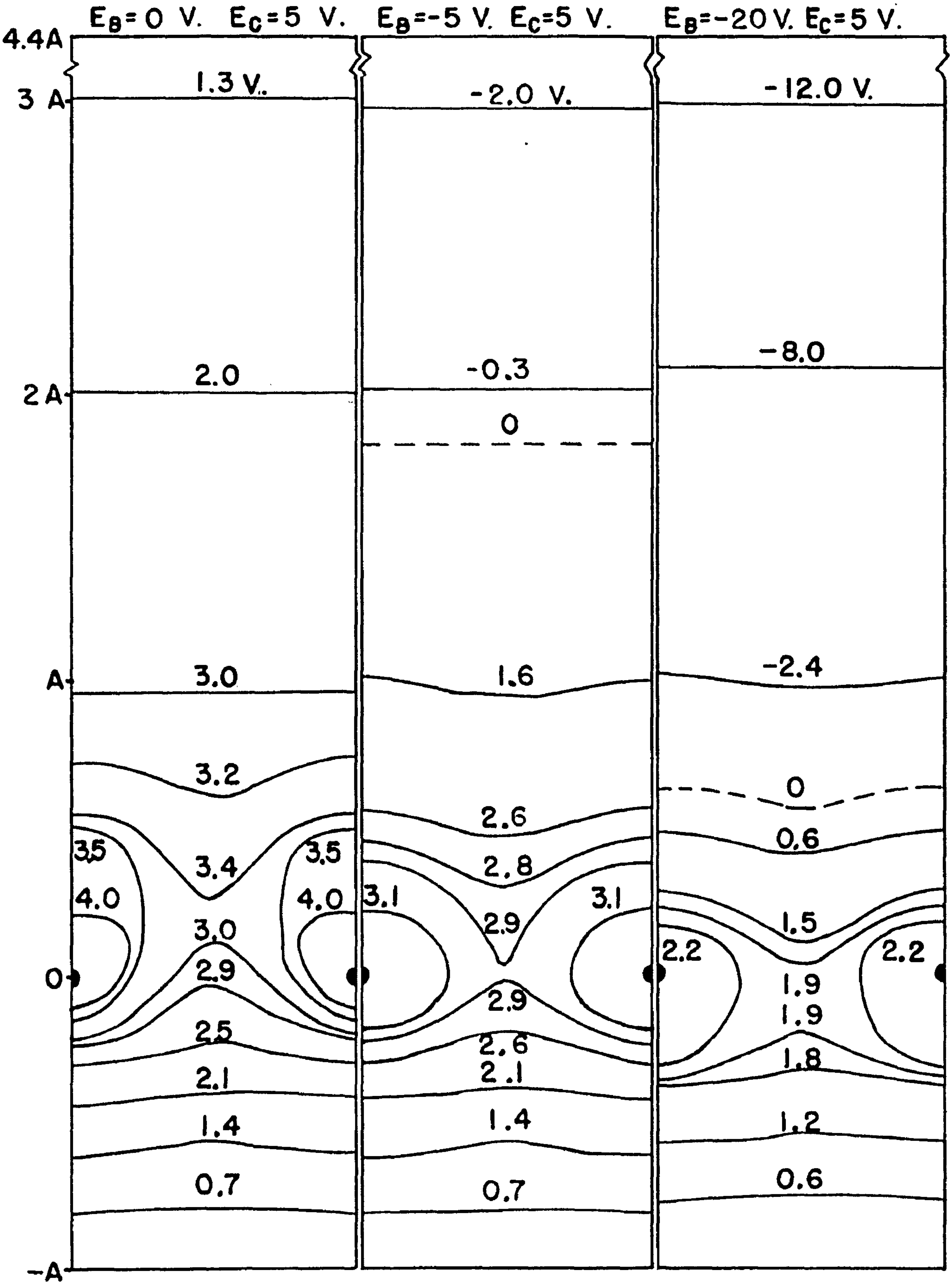

Fig. 5. Equipotential Lines in an Inverted Plane-electrode Triode.

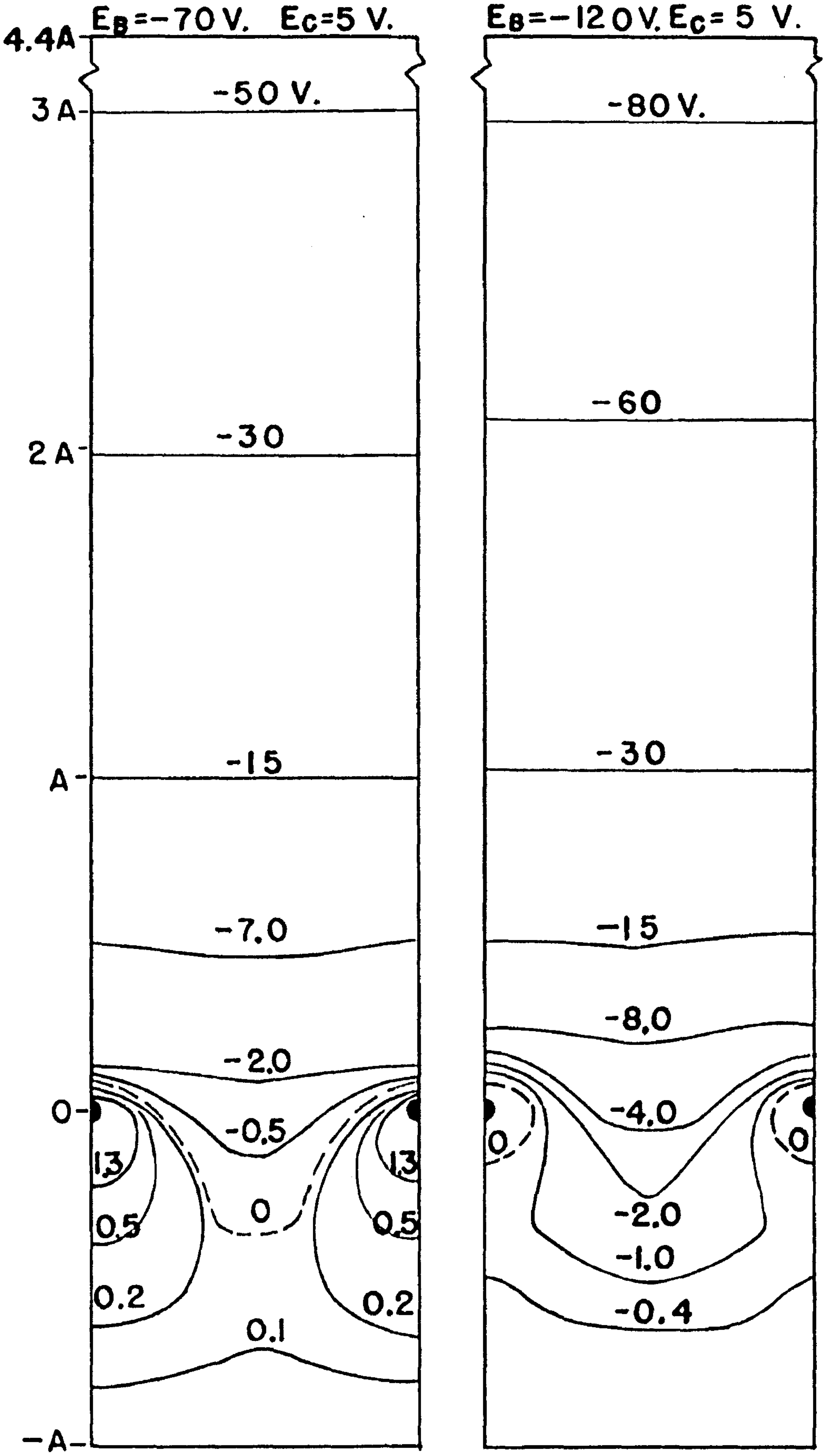

Fig. 6. Equipotential Lines in an Inverted Plane-electrode Triode.

the utilization of a triode in an inverted circuit is the amount of heat which the grid can dissipate, usually expressed in terms of maximum allowable power input to the grid. The ratings given by manufacturers for tubes in conventional circuits are of little help since these ratings presuppose that the plate must also dissipate heat. Furthermore, in normal operation, the power input to the plate is much greater than that to the grid. For most receiving type triodes designed for voltage amplifier service the ratings do not permit the grid to be driven positive, and the allowable grid dissipation rating is essentially zero. For power triodes the grid is usually permitted to go somewhat positive for a part of a cycle and the manufacturers' ratings permit some power input to the grid.

When a triode is used in an inverted amplifier, the plate is biased negatively so that even on the peaks of the signal voltage little or no plate current flows. The grid is held at a positive voltage, and grid current flows most or all of the time. It is to be expected that under this condition of operation the grid would be able to dissipate safely a small amount of heat, both by radiation to the plate which would be at a lower temperature, and by conduction along the grid leads. In order to get an estimate of the amount of power which could be handled by the grids of typical triodes in inverted operation, three types, a 6J5, a 6C5, and a 2022, were connected so that they each passed a grid current of 10 ma with a positive grid voltage of approximately 12 volts. The negative bias supplied to their plates was approximately 160 volts. The tubes were permitted to pass this amount of grid current until they reached a stable operating temperature. There were no perceptible changes in any voltages or currents during the period of the test, and all outward indications were that the tubes had not been damaged in any way. In order to make certain that the grids and other parts of the tubes had not been damaged the tubes were dismantled. There was no evidence of damage of any kind to any part of the tubes, even when inspected under a low-power magnifying glass. A 6N7 twin-triode power tube, which had passed grid currents of 20 to 30 ma during a preliminary investigation, was also dismantled. It, too, showed no evidence of damage. This tube is equipped with a grid radiator and is rated for a peak current of approximately 22 ma and no damage would be expected of this tube. As a result of these tests it was concluded that any one of these tubes could safely pass a grid current of 10 ma without damage. The validity of this conclusion, for a 6J5 at least, is demonstrated by the fact that in later investigations currents as high as 15 ma were passed by a 6J5 numerous times with no indication of changes in the characteristics of the tube.

2. Inverted static characteristic curves.

A preliminary investigation of the static characteristic curves of a 6J5, 6C5, 2C22, and 6N7 showed that the curves were all similar. Later checks on specific items verified this observation. Because of this similarity among the various types of tubes cited above it appeared unnecessary to obtain complete data on all of them and a type 6J5 was selected for which complete data were obtained. Two 6J5 tubes were used for most of the investigation. These two tubes were not specially selected, but did have approximately the same characteristics. One was used for obtaining preliminary data and in making certain the various circuits were working properly. The second tube was substituted when quantitative data were being recorded. This procedure was adopted to protect the second tube from inadvertent damage. There was, however, no perceptible change in either 6J5 tube during the course of the investigation.

The static characteristic curves were obtained in a conventional manner through the use of d-c voltmeters and milliammeters and adjustable direct current sources. It was found desirable, however, to obtain detailed data for small negative plate voltages in the case of the inverse transfer curves, in addition to the regular curves. In this region the curves possess two or three maxima and minima. In order to define clearly the exact position of these maxima and minima a 0-1 ma d-c milliammeter was connected in series with the 0-10 ma meter which was used to measure the grid current. Most of the grid current was bucked out by a 1.5-volt battery and 10,000-ohm rheostat in shunt with the 0-1 ma meter. This small meter could then be used to define clearly the maxima and minima points while the larger meter indicated the total current taken by the grid. This procedure had the effect of giving a magnification of ten times in these regions of interest.

Independent sets of data were obtained for the three static characteristic curves. The first presents grid current as a function of grid voltage for various constant values of negative plate voltage. This gives the family of grid characteristic curves. The second presents grid current as a function of negative plate voltage for various constant values of grid voltage. This gives a family of inverse transfer curves. The third presents grid voltage as a function of negative plate voltage for various constant values of grid current. The curves of this family are usually called the constant-current curves. Any one of the three families of curves contains the same data as the other two. However, each family is useful in interpreting certain aspects of a tube's behavior.

The grid characteristics for various constant values

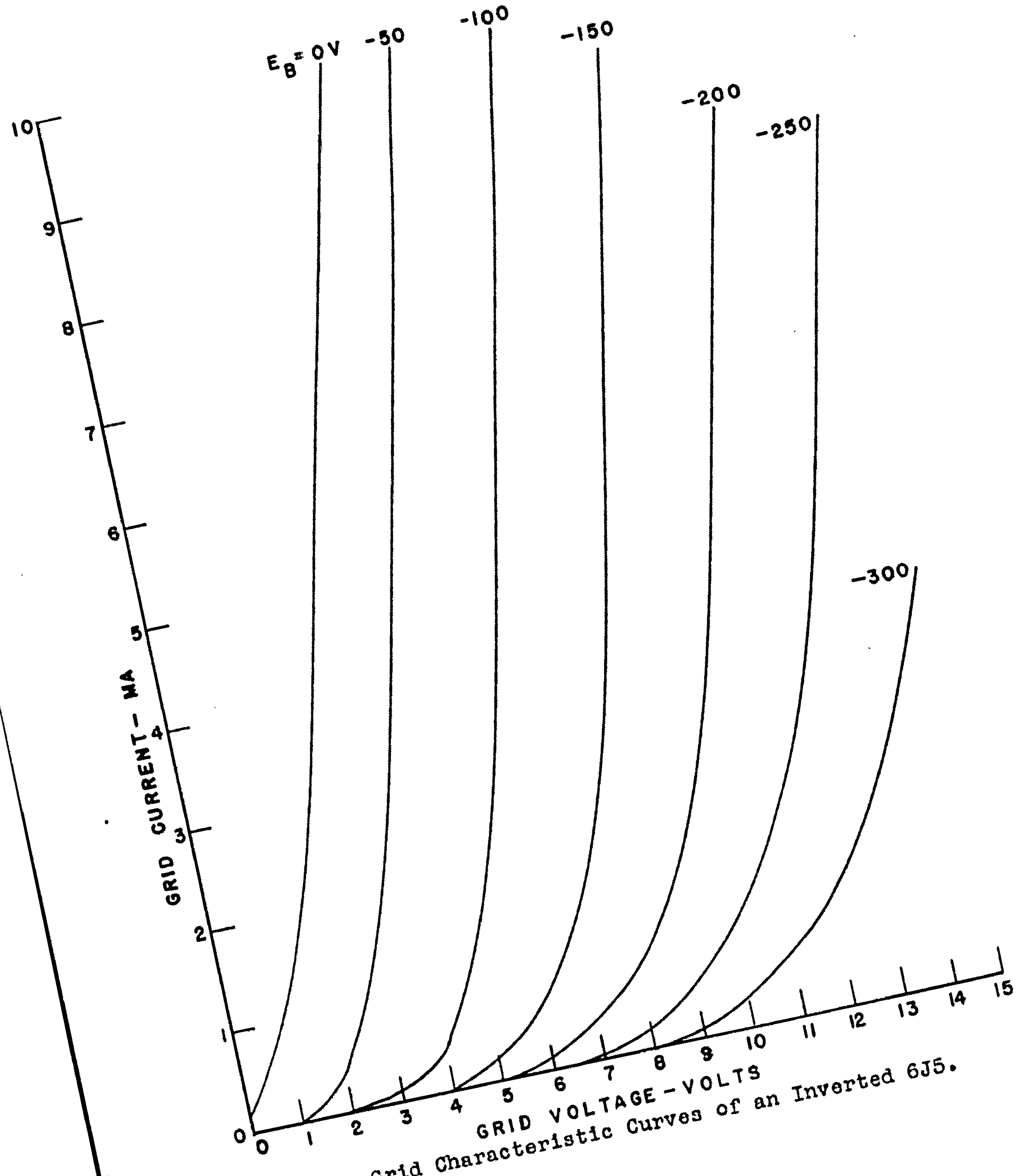

Fig. 7. Grid Characteristic Curves of an Inverted 6J5.

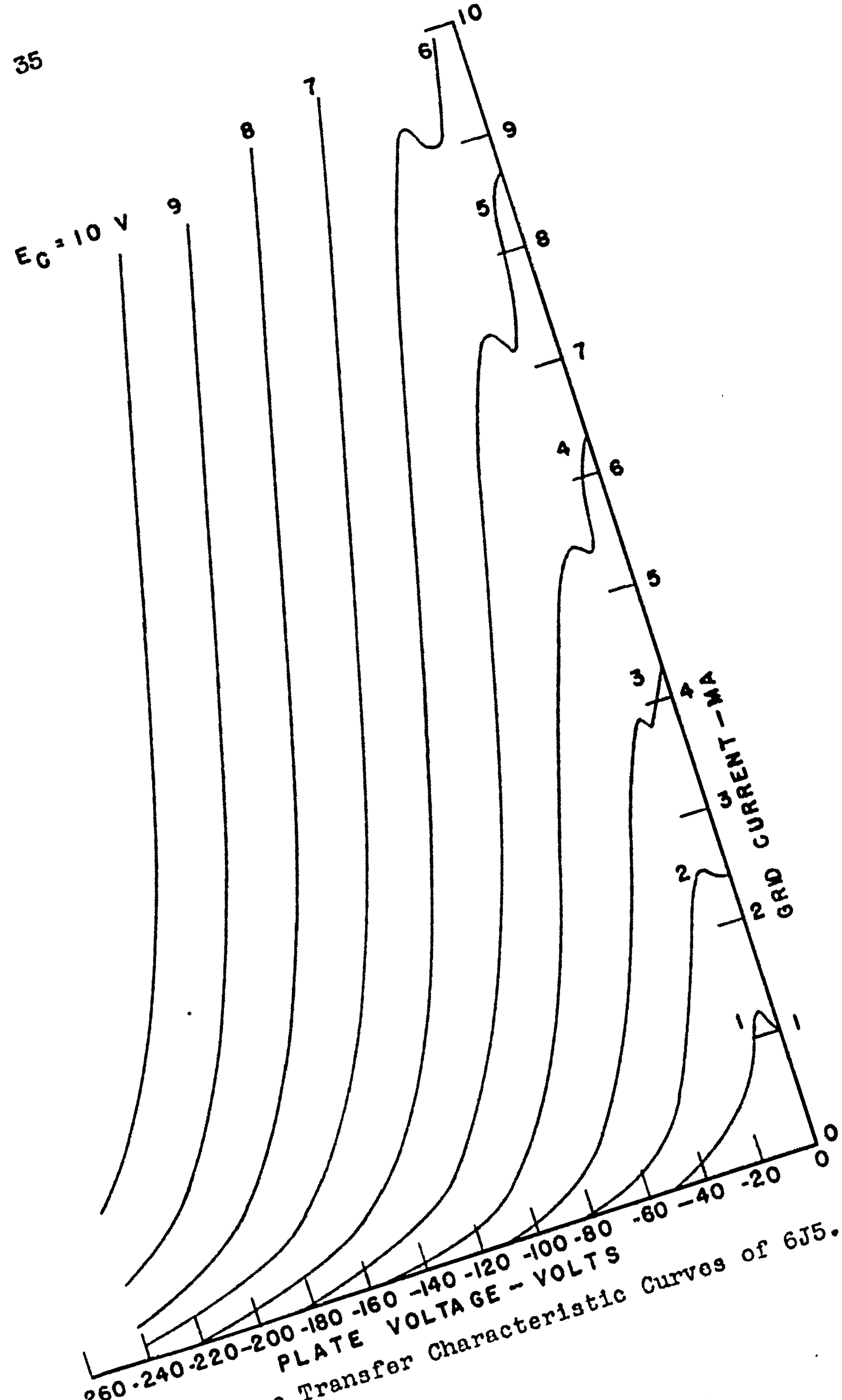

Fig. 8. Inverse Transfer Characteristic Curves of 6J5.

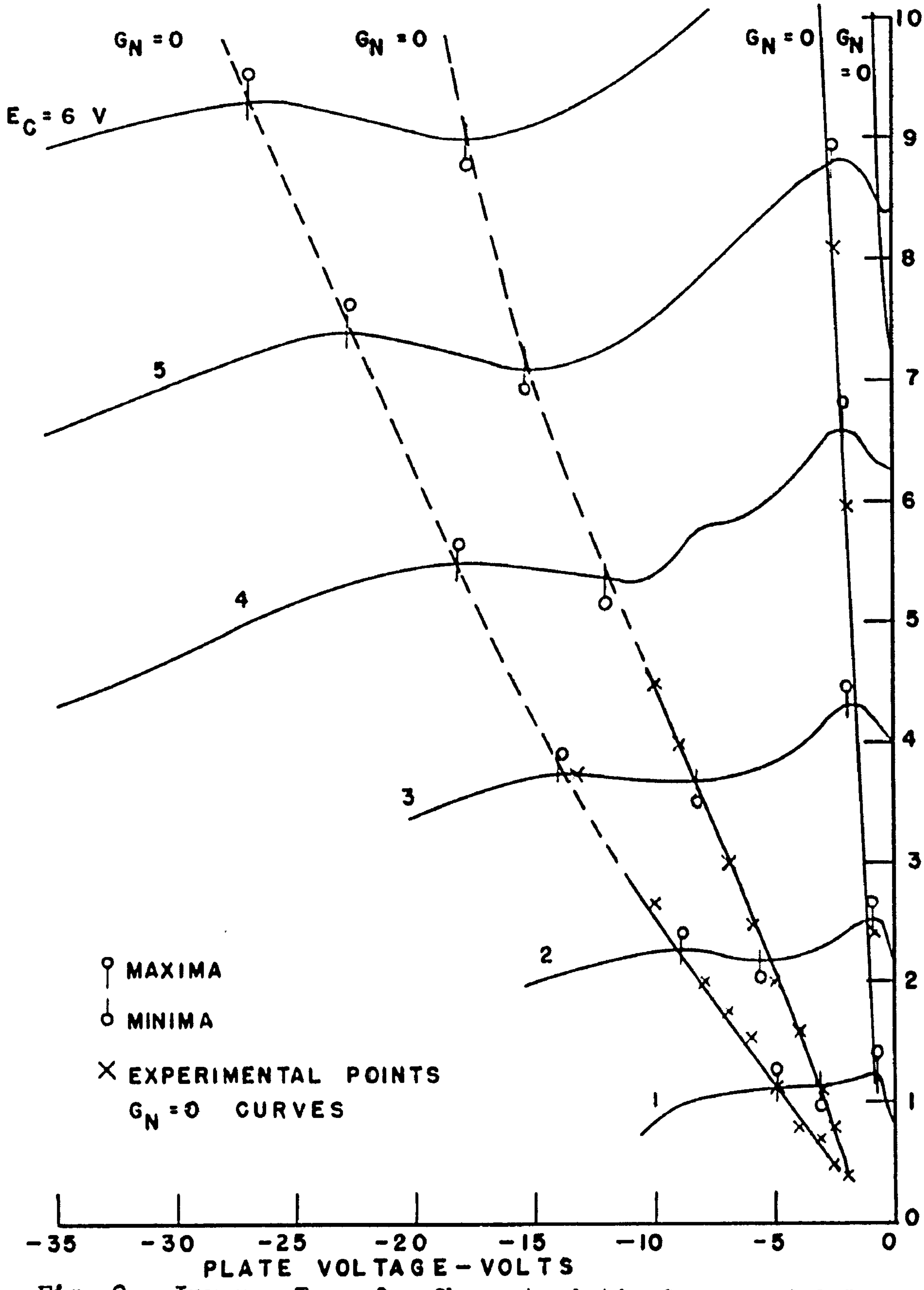

Fig. 9. Inverse Transfer Characteristic Curves of 6J5 for Small Negative Plate Voltages.

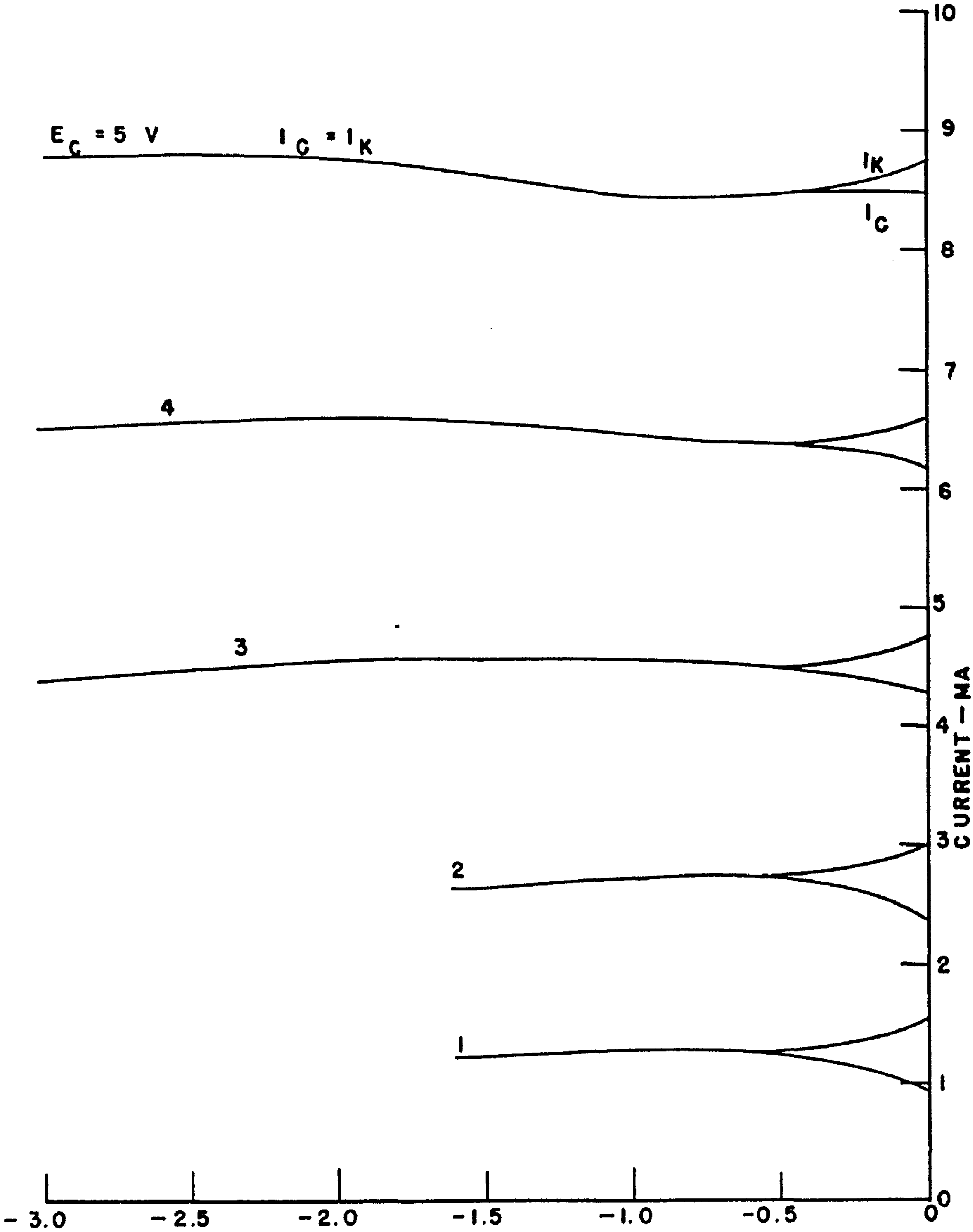

Fig. 10. Grid and Cathode Currents of 6J5 for Small Negative Plate Voltages.

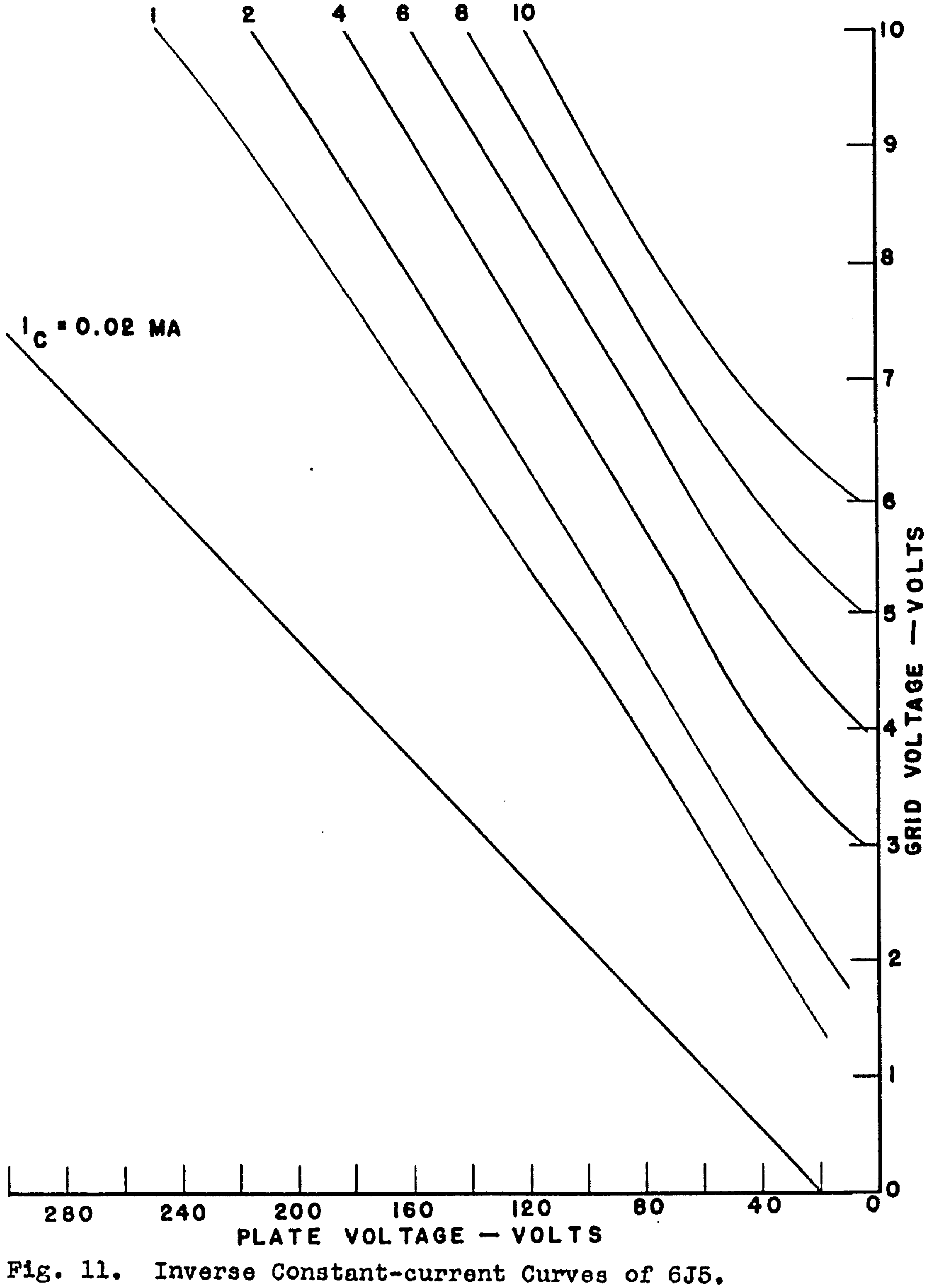

Fig. 11. Inverse Constant-current Curves of 6J5.

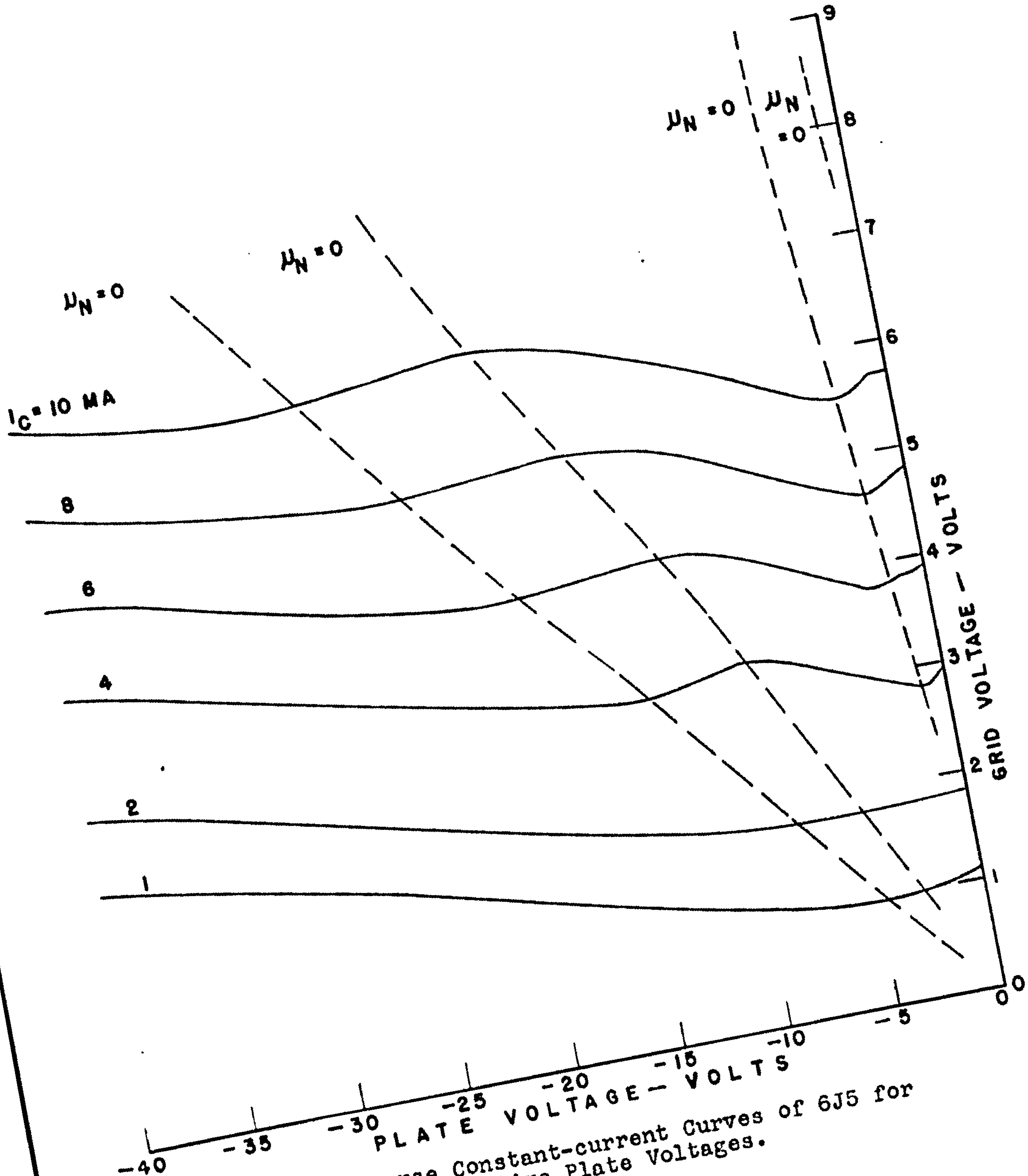

Fig. 12. Inverse Constant-current Curves of 6J5 for Small Negative Plate Voltages.

of negative plate voltage are shown in Fig. 7. These curves are quite regular and are similar to the ordinary plate characteristics of a triode. There are some irregularities which appear for low values of negative plate voltage. These irregularities are suggested by the closeness of the 0 and -50 volt curves compared to the spacing of the other curves of the family. The irregularities are best shown in the inverse transfer curves and constant-current curves and are discussed in connection with these curves.

The inverse transfer curves for a 6J5 are shown in Fig. 8. These curves are regular and are similar to the ordinary transfer characteristics of a triode except for low values of negative plate voltage. In this region the curves are highly irregular. A more detailed study of these irregularities is shown in Fig. 9. A further study was made of the behavior of the grid, and total (cathode) currents for small negative plate voltages. The data are shown as curves in Fig. 10.

The inverse constant-current curves are shown in Fig. 11. These curves are fairly regular. The increased spacing between curves as the current is reduced indicates a semi-remote cut-off characteristic. There are some irregularities in the curves for small negative plate voltages. These are shown in detail in Fig. 12.

3. Inverted-tube parameters.

The inverse amplification factor (μ_n), inverse mutual conductance (g_n), and dynamic grid resistance (r_g) were measured by the use of a General Radio Type 561-D Vacuum-Tube Bridge. This bridge is arranged so that a large range of values of any of the three tube parameters may be measured. In addition, a switch on the panel enables the operator to measure negative as well as positive quantities with equal facility. Before consistent results could be obtained it was found necessary to clean and lubricate all the contact points of the decade resistors which are a part of this bridge. It was also necessary to shunt the direct voltage sources for both the grid and plate with 10-microfarad capacitors to reduce 60-cycle noise to a level low enough that it was not troublesome. A filament transformer was used to supply filament power. There was no apparent difficulty from 60-cycle noise from this source. An audio oscillator, set at 1000 cps, was used as the signal source. The detector was a cathode ray oscilloscope.

All three of the tube parameters were measured for each setting of grid and plate voltage. The results were checked for discrepancies by use of the relation $\mu_n = g_n \times r_g$. All values checked to within 2 per cent, which is the rated accuracy of the bridge. Most values checked much better than this.

The measured values of the inverse amplification factor are plotted in Fig. 13. Except at small values of grid current, less than about 3 ma, the plate voltage has relatively little effect on the value of the amplification factor. The curves appear to level off at values between 0.040 and 0.045. This is an expected result since the amplification factor is primarily determined by geometrical considerations and to a lesser extent by the actual voltages applied to the electrodes. An important exception to the above statements is indicated by the -50-volt curve. This curve rises to a peak around 2 ma, then drops off rather rapidly. Curves for negative plate voltages smaller than about -50 volts behave erratically. In this region the amplification factor may be negative as well as positive. This peculiarity is discussed in connection with the analysis of the inverse transfer and constant-current curves.

The inverse mutual conductance curves are shown in Fig. 14. These curves are regular except at small negative values of plate voltage. The -50-volt curve suggests this irregularity. The behavior of the curves for smaller negative plate voltages is discussed along with the inverse transfer curves. The inverse mutual conductance may be negative as well as positive in this low voltage region.

The dynamic grid resistance curves are shown in Fig. 15.

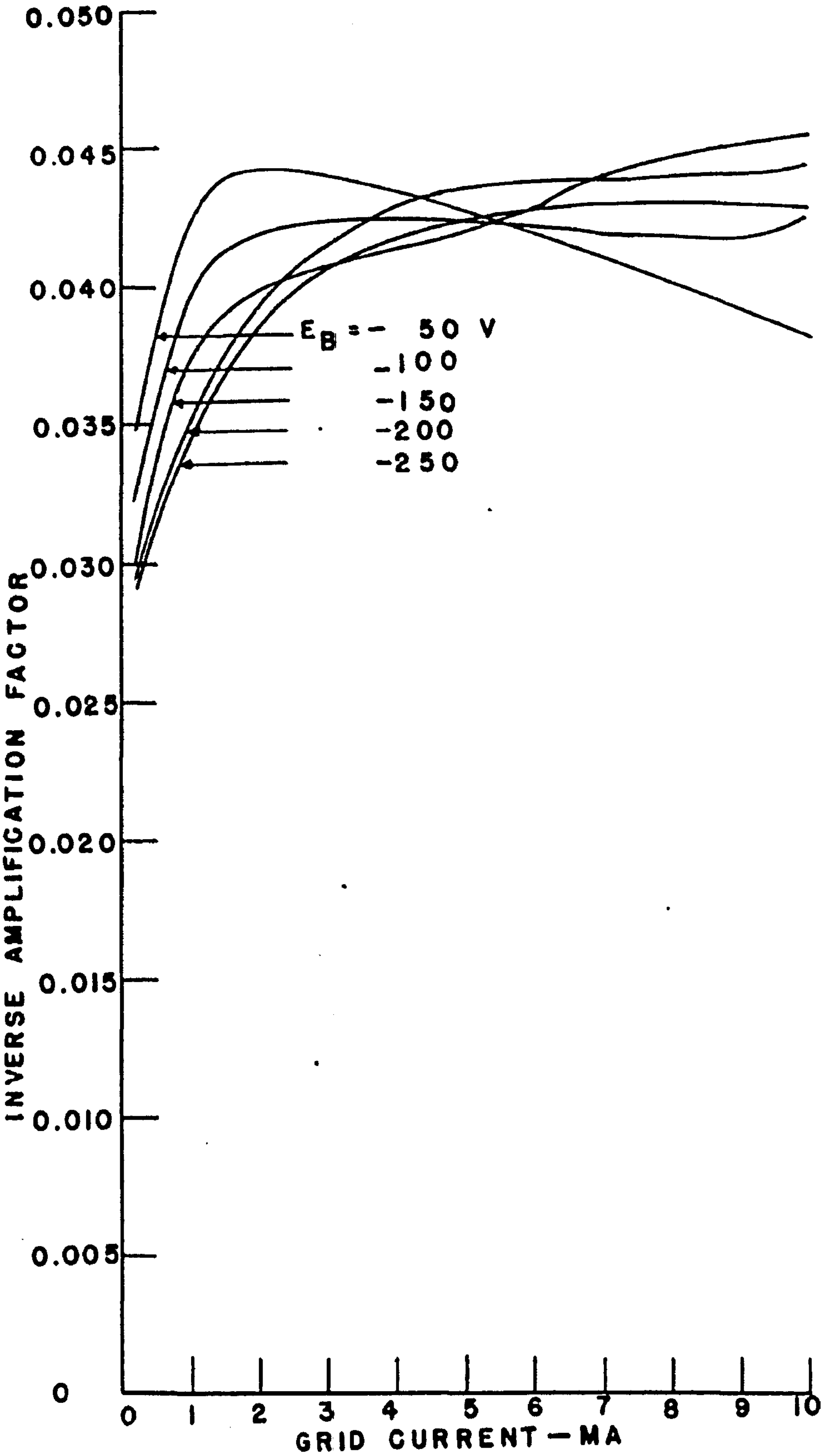

Fig. 13. Inverse Amplification Factor Curves of 6J5.

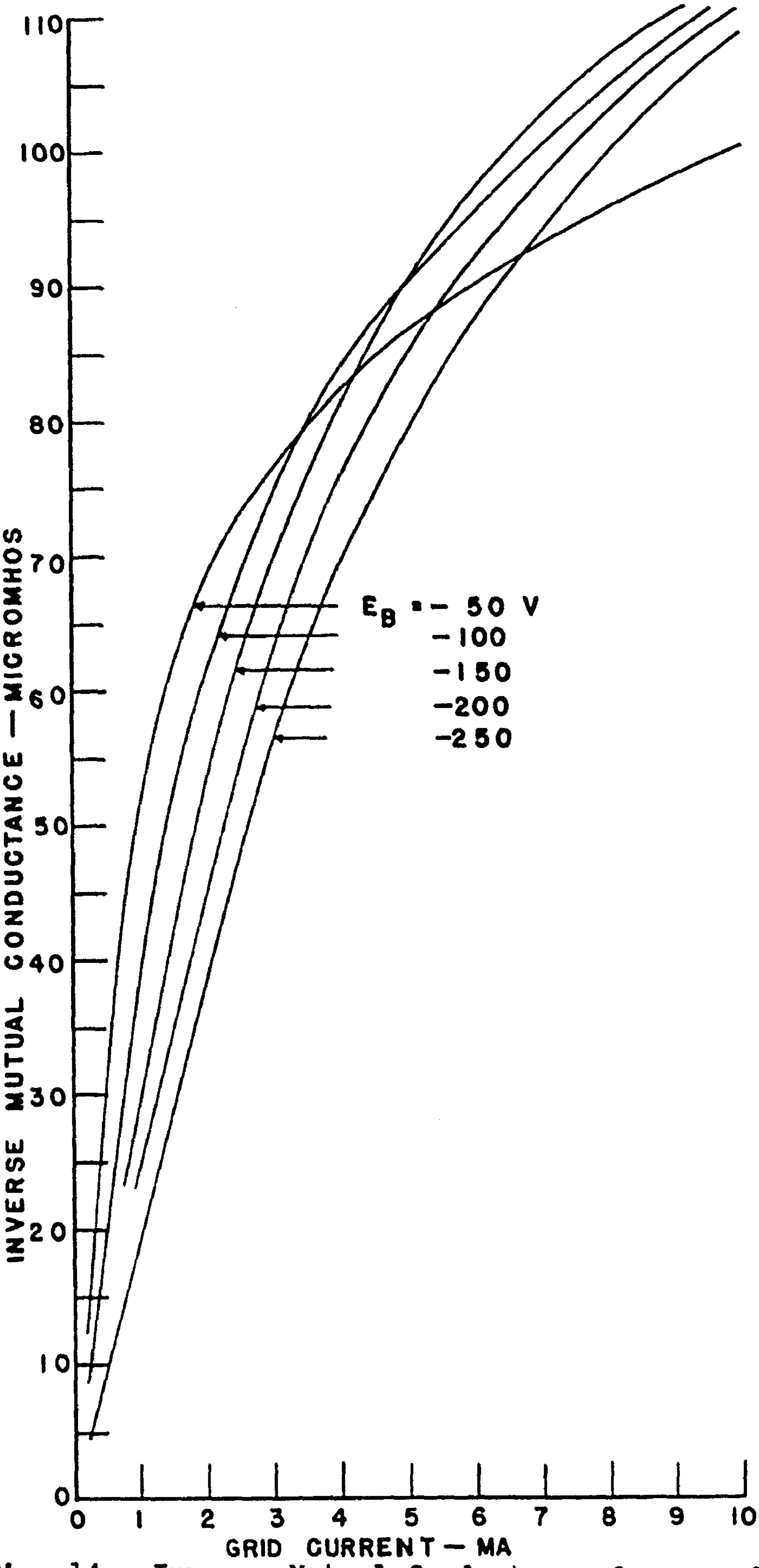

Fig. 14. Inverse Mutual Conductance Curves of 6J5.

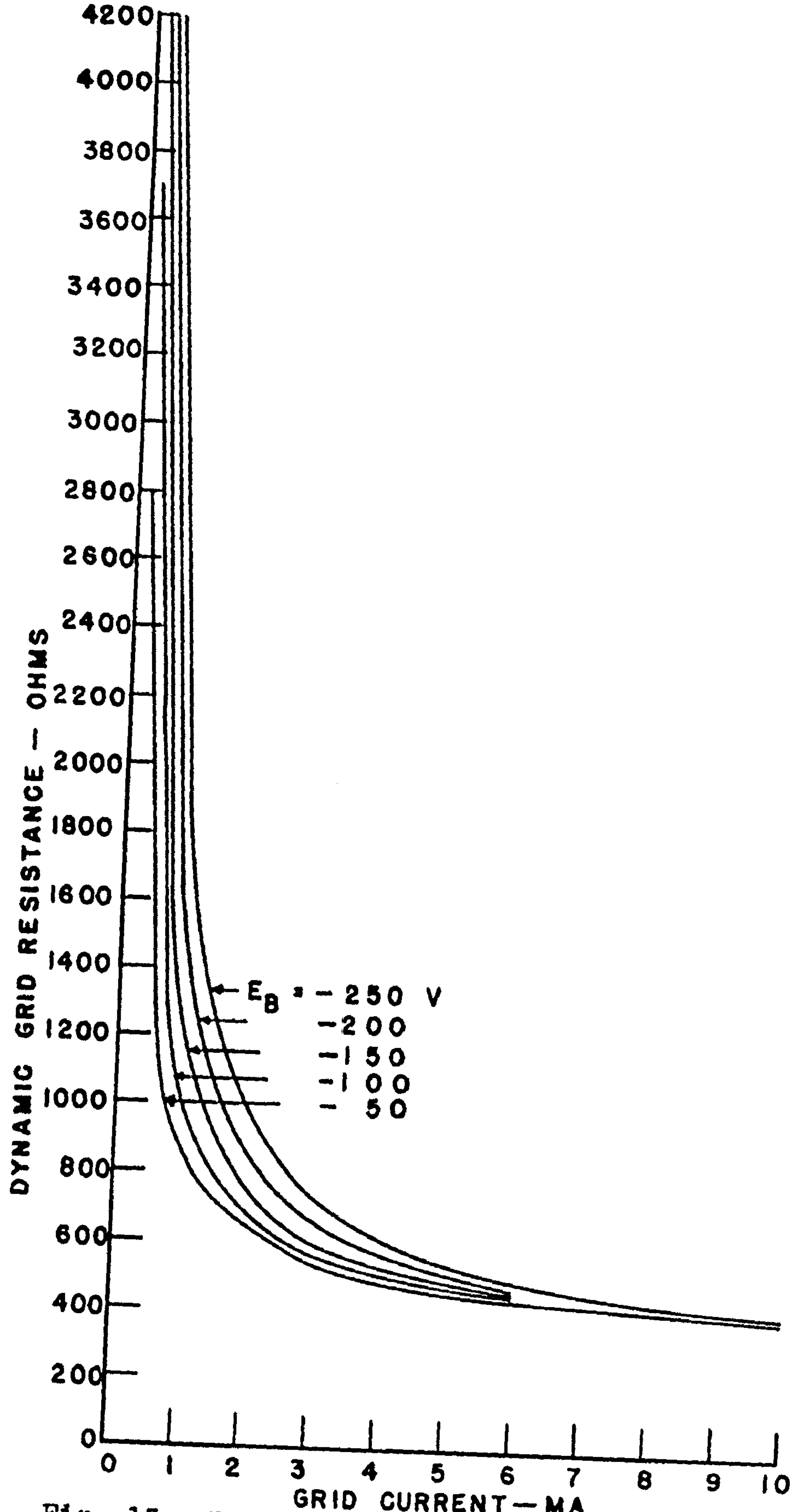

Fig. 15. Dynamic Grid Resistance Curves of 6J5.

For low values of grid current the resistance is quite high regardless of the negative plate voltage. This behavior is expected since the resistance must approach infinity as the grid current approaches zero. For grid currents in excess of about 5 ma the negative plate voltage has relatively little effect on the dynamic grid resistance. In this region the grid resistance is approximately 450 ohms. The resistance was positive over all the inverted region which could be investigated.

A supplementary set of curves of inverse amplification factor, inverse mutual conductance, and dynamic grid resistance are seen in Figs. 16, 17 and 18, respectively. These curves are for small negative values of plate voltage and show the peculiarities found in the curves in this region. All the curves of inverse amplification factor and inverse mutual conductance show a tendency to decrease after an initial increase for low values of grid current. The curves for plate voltages of -10 and -20 volts show a region in which the inverse amplification factor and the inverse mutual conductance are negative, followed by a region of positive values. The curves for a plate voltage of -30 volts show the beginnings of a region of negative values. There also is evidence of the start of a sharp drop in the -40 volt curves at the higher values of grid current. It was not feasible to investigate the behavior of the curves in

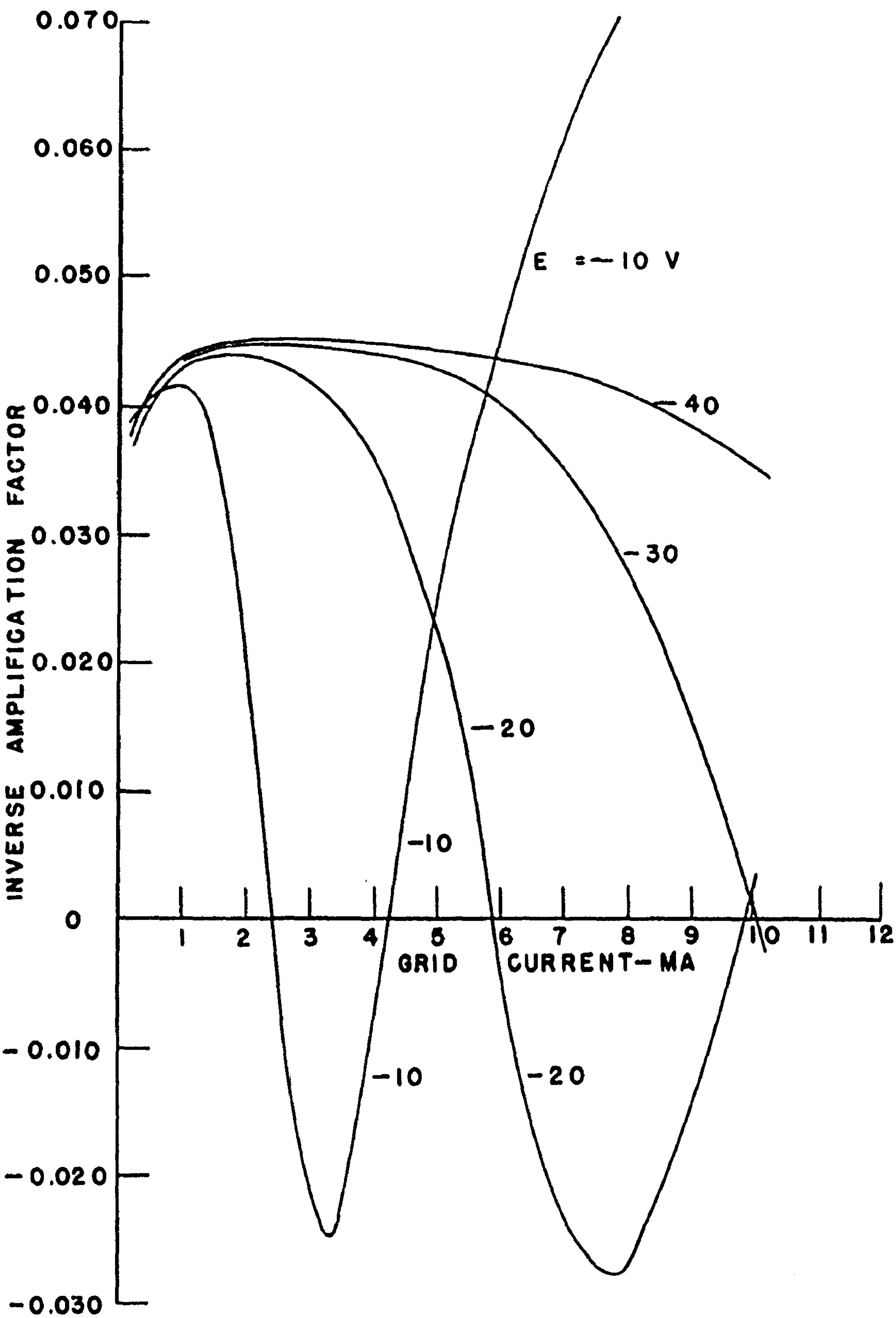

Fig. 16. Inverse Amplification Factor Curves of 6J5 for Small Negative Plate Voltages.

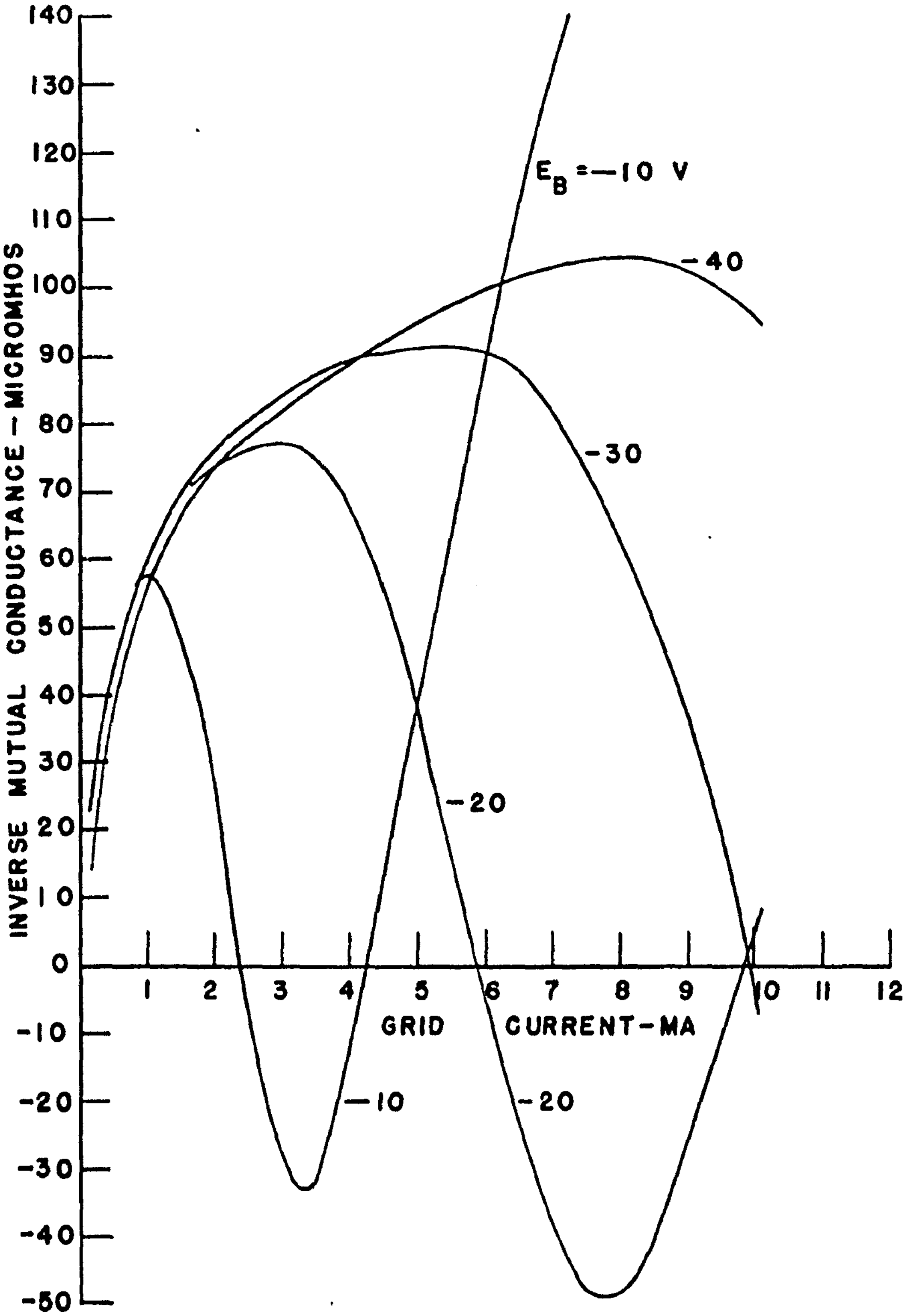

Fig. 17. Inverse Mutual Conductance Curves of 6J5 for Small Negative Plate Voltages.

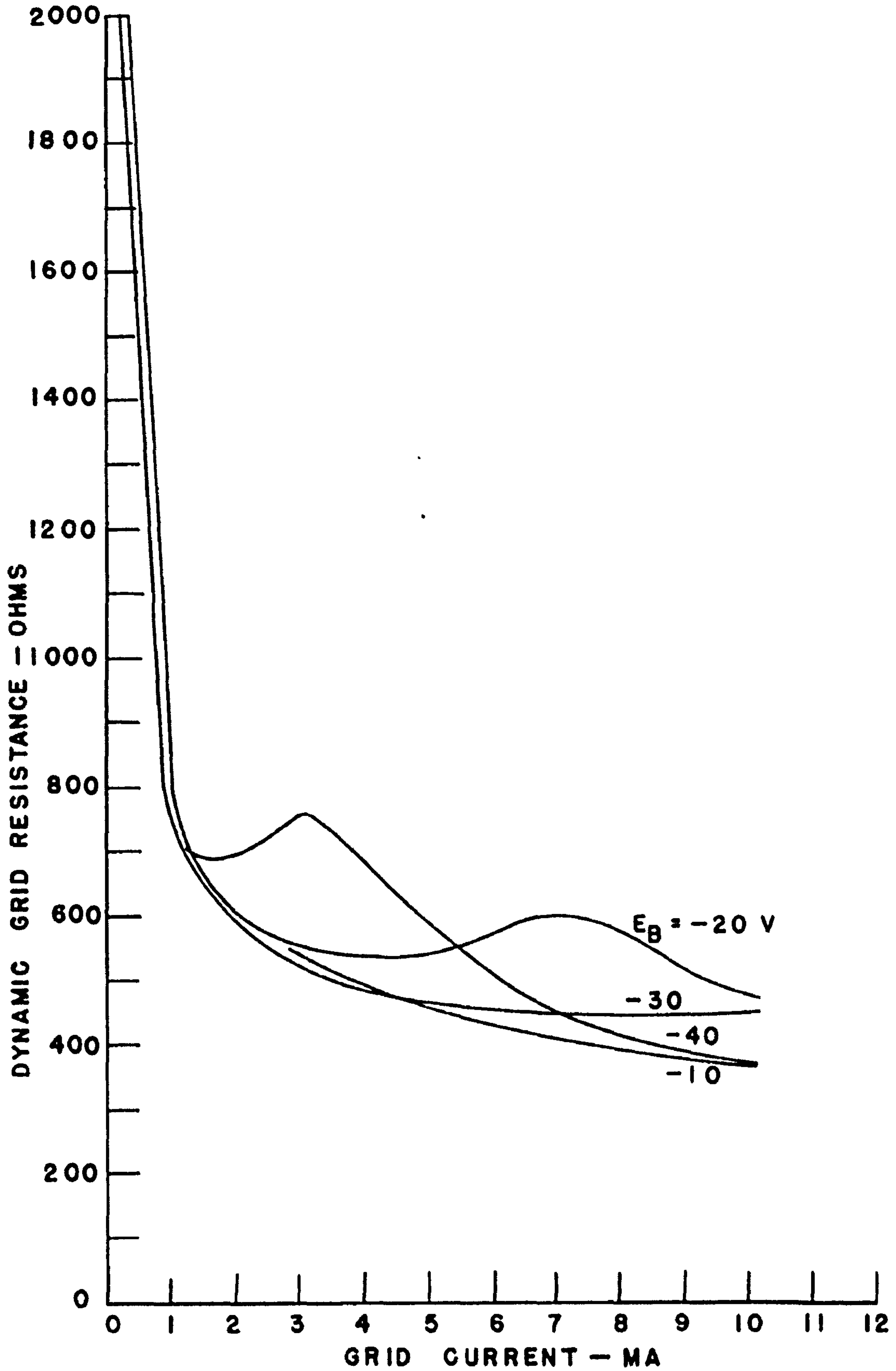

Fig. 18. Dynamic Grid Resistance Curves of 6J5 for Small Negative Plate Voltages.

the regions in which the grid current was greater than 10 ma because of the possibility of damage to the grid of the tube. The dynamic grid resistance curves of Fig. 18 are positive everywhere.

4. Gain and distortion.

The gain and distortion of the three types of inverted amplifiers were measured under experimental conditions. The load resistances were calibrated resistors. The alternating input and output voltages were measured with vacuum-tube voltmeters. The distortion was measured with a General Radio Type 736-A Wave Analyzer. In order to obtain a check on the measured value of gain, computations of gain were made using the derived expressions of Table 1 and values of the inverse amplification factor and dynamic grid resistance from Figs. 13 and 15. No effort was made to check the measured values of distortion by means of graphical analysis based on the characteristic curves since the observed values of distortion are too small for a check based on a graphical method to be of much use.

The experimental data for a plate-input grid-output amplifier are shown in Table 7. The measured and calculated values of gain agree to within one per cent in all cases except one. The second harmonic distortion rises with signal input until the peak of the signal drives the plate voltage into the region between -50 and 0 volts. In this region the second harmonic distortion drops sharply. In the same region the third harmonic distortion shows a sharp rise. The wave-form of the output voltage, as viewed on a cathode ray oscilloscope, also changes markedly in this region. This peculiar behavior results from the irregularities of the inverse transfer curve as seen in Fig. 8.

The experimental data for a plate-input cathode-output amplifier are shown in Table 8. The experimental values for gain are not much different than the values observed for the plate-input grid-output amplifier. The maximum deviation between measured and calculated values of gain is about two per cent. Both second and third harmonic distortion tend to rise as the signal voltage is increased. There were no sharp changes in the amount of distortion or the appearance of the output voltage wave as were observed in the case of the plate-input grid-output amplifier.

The experimental data for a cathode-input grid-output amplifier are shown in Table 9. In this circuit it was necessary to adjust the direct voltages in the grid and plate circuits so that the direct voltage from grid to cathode of the tube had the proper value. The values of the direct grid voltage (E_c) for various load resistances are shown in the table. The measured value of gain approached unity as the size of the load resistor was increased. The maximum deviation between measured and calculated values of gain is about 3.5 per cent. This deviation is somewhat higher than that of the other two inverted amplifiers. Both second and third harmonic distortion rise with signal voltage and decrease as the load resistance is increased. The maximum input voltage (rms) which could be supplied to the circuit without excessive distortion was 10 volts for load resistances of 10,000 ohms and greater. The input signal was restricted to 4 volts or less when the load resistance was 1000 ohms.

Table 7

Plate-input Grid-output Amplifier*
Gain and Distortion

f = 1000 cps
E_{bb} = -150 volts
E_{cc} = 22 volts

	Load Resistance (R_L)					
	3500 Ohms			5000 Ohms		
I_c (Ma)	3.8	3.8	3.8	2.6	2.6	2.6
E_s (Volts)	60	80	100	60	80	100
E_o (Volts)	2.25	3.0	3.7	2.3	3.05	3.8
Gain (Meas.)	0.0375	0.0375	0.0370	0.0383	0.0381	0.0380
Gain (Calc.)	0.0372	0.0372	0.0372	0.0378	0.0378	0.0378
Distortion						
% 2nd	1.0	1.3	0.38	1.6	2.3	1.8
% 3rd	0.2	0.4	2.4	0.54	0.68	2.2

*See Fig. 2 for circuit. The tube was a Type 6J5 triode.

Table 8

Plate-input Cathode-output Amplifier*
Gain and Distortion

f = 1000 cps
E_{bb} = -150 volts
E_{cc} = 44.8 volts

	Cathode Resistance (R_K)					
	10,000 Ohms			50,000 Ohms		
I_o (Ma)	3.5	3.5	3.5	0.8	0.8	0.8
E_s (Volts)	60	80	100	60	80	100
E_o (Volts)	2.36	3.23	3.90	2.03	2.70	3.40
Gain (Meas.)	0.0394	0.0392	0.0390	0.0338	0.0338	0.0340
Gain (Calc.)	0.0386	0.0386	0.0386	0.0340	0.0340	0.0340
Distortion						
% 2nd	0.78	1.05	1.5	2.2	2.9	3.7
% 3rd	0.30	0.39	0.44	0.22	0.39	0.52

*See Fig. 3 for circuit. The tube was a Type 6J5 triode.

Table 9

Cathode-input Grid-output Amplifier*
Gain and Distortion

f = 1000 cps
E_2 = 200 Volts

	Load Resistance (R_L)								
	1,000 Ohms			10,000 Ohms			50,000 Ohms		
E_1 (Volts)	182	182	182	174	174	174	166	166	166
E_c (Volts)	10	10	10	8	8	7.5	6	6	5.9
I_c (Ma)	3.6	3.7	3.8	1.4	1.4	1.5	0.5	0.5	0.5
E_s (Volts)	2	3	4	2	4	10	2	4	10
E_o (Volts)	1.33	2.0	2.6	1.95	3.9	9.4	2.0	4.0	9.8
Gain (Meas.)	0.655	0.667	0.650	0.976	0.976	0.940	1.00	1.00	0.98
Gain (Calc.)	0.662	0.662	0.662	0.944	0.944	0.944	0.975	0.975	0.975
Distortion									
% 2nd	2.6	4.5	7.3	0.46	1.1	4.4	0.29	0.59	2.2
% 3rd	0.40	1.05	2.1	0.04	0.18	1.7	---	0.04	0.70

*See Fig. 4 for circuit. The tube was a Type 6J5 triode.

IV. DISCUSSION

A. Gain, Input Admittance, and Output Admittance

1. Comparison of inverted and conventional amplifiers.

A comparison of the voltage gain, input admittance, and output admittance of the inverted amplifiers and their conventional amplifier analogs is of interest. Tables 1 through 6 are referred to in the following discussion.

The gain of a plate-input grid-output amplifier, in the low frequency case, approaches a maximum equal to the inverse amplification factor for large values of load resistance while the gain of a plate-loaded amplifier reaches a maximum equal to the normal amplification factor. Since the inverse amplification factor is approximately the reciprocal of the normal amplification factor the gains of the two amplifiers are also approximately reciprocal. A triode with a large amplification factor would, in an inverted amplifier, yield a very small gain. For example, a 6J5 which has an amplification factor of 20 yields a maximum gain of approximately 0.04 in a plate-input grid-output amplifier.

The input admittance of a plate-input grid-output amplifier is ordinarily much lower than that of a plate-loaded amplifier, and is approximately equal to the susceptance of the plate-cathode plus the grid-plate interelectrode capacitances. The amount of gain in the circuit, which is ordinarily small in magnitude in respect to unity, has little effect on the size of the input admittance. Thus, the Miller effect[1], which is of considerable importance in the plate-loaded amplifier, is of little consequence in the plate-input grid-output amplifier. In the plate-loaded amplifier the grid-plate capacitance is magnified by the term (1 - A) where A is the gain. The gain of a plate-loaded amplifier with a medium mu tube such as a 6J5 is of the order of 15 and the phase angle is ordinarily approximately 180 degrees. The magnification of the grid-plate capacitance is approximately 16 times. In addition, if the phase angle is not exactly 180 degrees there is a positive or negative conductance component of the input admittance.

The output admittance of a plate-input grid-output amplifier, in the low frequency case, is equal to the dynamic grid conductance and that of the plate-loaded amplifier is equal to the dynamic plate conductance. The grid conductance of a triode is greater than its plate conductance. Thus, the output admittance of a plate-input grid-output amplifier is higher than that of a plate-loaded amplifier using the same tube. In terms of output impedance, the plate-input grid-

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[1]Ryder, op. cit., p. 205.

output amplifier yields the lower value. In the case of amplifiers using a type 6J5 triode the dynamic grid resistance is approximately 1/12 the dynamic plate resistance.

The gain of a plate-input cathode-output amplifier, in the low frequency case, approaches the value of the inverse amplification factor as a maximum. The gain of the cathode-follower approaches unity as a maximum. Thus the gain of the plate-input cathode-output amplifier is considerably smaller than that of the cathode-follower amplifier.

The input admittance of the plate-input cathode-output amplifier is approximately equal to the susceptance of the grid-plate and plate-cathode capacitances in parallel. The gain, which is small in respect to unity, has little effect on this admittance. In the case of the cathode-follower the gain is approximately unity and has a phase angle near zero in the usual circuit. This factor makes the effect of the grid-cathode capacitance almost negligible and the input admittance is primarily that of the grid-plate capacitance. For many triodes the three interelectrode capacitances are nearly equal. Thus, the input admittance of the plate-input cathode-output amplifier is approximately twice that of the cathode-follower.

The output admittance (or impedance) of the plate-input cathode-output and cathode-follower amplifiers is approximately the same because the sum of the grid conductance and inverse mutual conductance for typical triodes is approximately the same as the sum of the plate conductance and the mutual conductance.

The gain of the cathode-input grid-output amplifier, in the low frequency case, approaches unity as a maximum. The gain of the grounded-grid amplifier approaches $\mu + 1$ as a maximum. Thus the gain of the grounded-grid amplifier is considerably larger. Because the gain of the grounded-grid amplifier is the larger, the input admittance of this amplifier is also larger than that of a cathode-input grid-output amplifier. This is so because of the manner in which the gain enters the expressions for input admittance and the fact that the grid-cathode and plate-cathode capacitances of the ordinary triode are not much different. The input admittance of both amplifiers is ordinarily much higher than for the four amplifiers discussed previously.

The output admittance of the cathode-input grid-output amplifier tends to be higher than that of the grounded-grid amplifier. In the case in which the frequency is low enough that the interelectrode capacitances may be neglected and the source impedance is negligible, the output admittance of the cathode-input grid-output amplifier is equal to the dynamic grid conductance while that of the grounded-grid amplifier is equal to the dynamic plate conductance. With high source impedances the output admittance is small. Thus, the output impedances increase with the source impedance. The output and source impedances are not, however, linearly related.

2. Typical numerical values.

Gain, input admittance, and output admittance of the three inverted amplifiers under typical operating conditions are shown in Table 10. The same values of dynamic grid resistance (500 ohms), inverse mutual conductance (85 micromhos) and load resistance (5000 ohms) are assumed for the three amplifiers. Each of the three interelectrode capacitances is taken as 6$\mu\mu$f. This value of capacitance includes about 2.5 $\mu\mu$f for socket and wiring capacitance.[1] The calculations are made at angular frequencies of 1×10^6 and 10×10^6 radians per second. These values correspond to 0.159 and 1.59 megacycles, respectively. Gain is calculated at these frequencies and also for the low-frequency case in which the effect of the interelectrode capacitances may be neglected. In Table 10, the input and output admittances have been reduced to their equivalent values of shunt resistance and shunt capacitance. In the case of output admittance these

[1] Input admittance of receiving tubes. RCA application note AN-118. Harrison, New Jersey, Radio Corporation of America. April 15, 1947.

Table 10

Calculated Values of Voltage Gain, Shunt Input Resistance and Capacitance, Shunt Output Resistance and Capacitance, and Power Gain of Inverted Amplifiers

r_g = 500 Ohms
g_n = 85 Micromhos
$C_{gp} = C_{gk} = C_{pk} = 6\ \mu\mu f$
$Z_L = Z_K = 5000$ Ohms

	Plate-input Grid-output	Plate-input Cathode-output	Cathode-input Grid-output
Gain			
Low Freq	$-0.0386\ \angle 0^\circ$	$0.0372\ \angle 0^\circ$	$0.947\ \angle 0^\circ$
0.159 Mc	$-0.0386\ \angle -4.34^\circ$	$0.0372\ \angle 4.05^\circ$	$0.947\ \angle -1.14^\circ$
1.59 Mc	$-0.0472\ \angle -38.28^\circ$	$0.0455\ \angle 33.72^\circ$	$0.947\ \angle -1.45^\circ$
R_{in} (Ohms)[a]			
0.159 Mc	57 Meg	64 Meg	5280
1.59 Mc	570,000	661,000	5240
C_{in} ($\mu\mu f$)[a]			
0.159 Mc	12.17	11.58	7.92
1.59 Mc	12.22	11.77	11.2
R_{out} (Ohms)[a,b]			
0.159 Mc	500	---	---
1.59 Mc	500	437	500
C_{out} ($\mu\mu f$)[a,b]			
0.159 Mc	12	---	---
1.59 Mc	12	12	12
Rout (Ohms)[a,c]			
0.159 Mc	---	---	---
1.59 Mc	489	446	546,000
Cout ($\mu\mu f$)[a,c]			
0.159 Mc	---	---	---
1.59 Mc	9	10.15	11.75
Power Gain (db)			
0.159 Mc	12.30	12.48	-0.236
1.59 Mc	-5.95	-5.64	-0.269

[a]Shunt values.
[b]Zero source impedance.
[c]Infinite source impedance.

values have been calculated for the limiting cases of a signal source with zero and infinite internal impedances.

The gain of the plate-input grid-output amplifier is seen to increase with frequency. This result arises from the manner in which the interelectrode capacitance terms add, in a vector sense, to the other terms and can be interpreted as being the result of a limited amount of positive feedback. A similar result is noted for the plate-input cathode-output amplifier. The high- and low-frequency gain of the cathode-input grid-output amplifier is the same except for a slight phase shift in the high-frequency case.

The magnitude of the gain of both the plate-input grid-output and plate-input cathode-output amplifiers approaches a limiting value at high frequencies equal to $C_{gp}/(C_{gp} + C_{gk})$. For most triodes this limiting value will be approximately 0.5.

The gain of the cathode-input grid-output amplifier remains at 0.947 up to frequencies at least as high as 1.59 mc. The gain drops at higher frequencies toward a limiting value of $C_{gk}/(C_{gk} + C_{gp})$.

The shunt input resistance of the plate-input grid-output and the plate-input cathode-output amplifiers is quite high, and is well above one-half megohm at the operating frequency. The shunt input resistance of the cathode-input grid-output amplifier is low and is not much greater than the load resistance. The shunt input capacitance is nearly the same in all three cases.

The shunt output resistance is nearly the same and approximately equal to the dynamic grid resistance in all cases except one. The exception is the case of the cathode-input grid-output amplifier supplied from a source with an infinite impedance. In this case the shunt output resistance is 546,000 ohms. This amplifier has a low shunt output resistance when the source impedance is low. The results, therefore, indicate that the output resistance depends to a large extent on the source impedance, and that the shunt resistance is large when the source impedance is large. Thus, it would be expected that the output shunt resistance would approach the dynamic grid resistance of the inverted triode when it was driven by another triode, but that this resistance would assume rather large values when driven by a pentode amplifier.

The shunt output capacitance is almost the same in all cases. The only case in which there is a significant difference in the value of the output capacitance is that of the plate-input grid-output amplifier supplied by a high impedance source. In this case the shunt output capacitance is 9 $\mu\mu$f compared to about 12 $\mu\mu$f in the other cases.

B. Minima and Maxima of Characteristic Curves

1. Contours of $g_n = 0$ and $\mu_n = 0$.

Peculiarities in the inverse transfer and constant-current curves in regions of small negative plate voltage have already been mentioned. The nature of these peculiarities is best seen in Figs. 9 and 12 which show the detailed behavior of these curves.

Contours of $g_n = 0$ are shown on the curves of Fig. 9. The points of intersection of these contours with the transfer curves correspond, in practically all cases, to measured points of minimum or maximum current. Any deviations must be attributed to experimental inaccuracies. Regions in which the slope of the transfer curves is negative represent negative inverse mutual conductance. Values of negative inverse mutual conductance in these regions were readily measured by means of the vacuum-tube bridge. No contours of negative inverse mutual conductance are shown since they fall close to the contours for zero inverse mutual conductance and tend to confuse the curves. For the same reasons contours of positive inverse mutual conductance are not shown in Fig. 9 although such contours are readily obtained by the use of the vacuum-tube bridge.

Contours of zero inverse amplification factor are shown in Fig. 12. These contours cross the constant-current curves at either maximum or minimum points. Regions of positive slope of the constant-current curves represent regions of negative inverse amplification factor. Two regions of positive slope are shown on Fig. 12. Regions in which the amplification factor is negative are readily found by the use of the vacuum-tube bridge. Contours of negative inverse amplification factor are not shown in Fig. 12 in order to keep the presentation comparatively simple.

The manner in which the slope of the inverse transfer curves and of the constant-current curves varies can be seen in Figs. 17 and 16,respectively. The inverse mutual conductance curves of Fig. 17 represent the slope of the inverse transfer curves of Figs. 8 and 9. The regions of negative slope, i.e., negative inverse mutual conductance, are seen to occur for negative plate voltages smaller than about 40 volts, although the -40-volt curve shows a downward trend which may result in a negative inverse mutual conductance at values of grid current greater than 10 ma, which was considered to be the safe value of maximum grid current for the tube.

The inverse amplification factor curves of Fig. 16 represent the negative of the slope of the curves of Figs. 11 and 12. The curves of Fig. 16 are similar in shape to those of Fig. 17.

The variations of dynamic grid resistance in regions of low negative plate potentials are seen in Fig. 18. The dynamic grid resistance is positive over the entire region shown, and there was no experimental indication that there would be negative values anywhere in the inverted region. Because the dynamic grid resistance is always positive, the regions in which the inverse amplification factor and the inverse mutual conductance are negative must correspond. This is true because of the relation $\mu_n = g_n \times r_g$ which must hold at all possible operating points.

2. Reasons for minima and maxima.

The minima and maxima of the inverse transfer and constant-current curves and the regions of zero and negative inverse mutual conductance and amplification factor are all related. The different methods of presentation of these characteristics merely serve to emphasize various features of the general behavior of the tube. For this reason the inverse transfer curves of Fig. 9 will be referred to in the following discussion. The discussion can, however, be applied quite generally to the constant-current curves of Fig. 12 or to the curves of inverse mutual conductance and inverse amplification factor.

Before proceeding to a detailed discussion of the reasons underlying the peculiarities of the various characteristics of the 6J5 tube in regions of small negative plate voltage it should be noted that similar behavior was observed for the following triode types: 6C5, 2C22, 6N7, and Western Electric 223-A. The first two types of triodes have overall electrical characteristics nearly the same as those of the 6J5. They both employ cylindrical cathode, grid and plate structures as does the 6J5. The 6N7 is a twin triode intended for power amplifier service. Each of the triode sections has an amplification factor of 35 and a dynamic plate resistance of about 11,000 ohms. The mechanical construction of the 6N7 is similar to that of the 6J5 except for closer spacing of the grid wires and a carbonized coating on both the grid and plate to permit these electrodes to radiate their heat more effectively. The grid also has a radiating flag which extends above the plate structure. The Western Electric 223-A is quite different from any of the other tubes. Its grid and plate are essentially parallel planes. The tungsten filament is w-shaped. The amplification factor is approximately 2.5 and the dynamic plate resistance is approximately 2000 ohms.

A possible explanation for the minima in the inverse transfer curves of the 6J5 would be that they are caused by secondary emission effects. However, this explanation is found upon examination not to be valid since secondary emission requires the presence of a positive collection electrode other than that from which secondary electrons are being emitted if the electrode suffering secondary emission is to exhibit a decrease of current. Such an electrode is not present in the inverted triode since the plate is held at a negative potential.

A better explanation for the peaks and dips, suggested by Van der Pol[1], is given by consideration of the electric field configuration and space charge distribution within the tube at small negative plate potentials. Van der Pol does not go into detail in his explanation so the following discussion is an extension of Van der Pol's remarks.

Most of the electrons which leave the cathode go to the grid. Some electrons, however, penetrate into the grid-plate region and have slightly more than enough energy to reach the plate because of their emission velocity which is equivalent to about 0.6 electron volt for the most energetic electrons. As the plate voltage is made slightly more negative the electrons which penetrate the grid-plate space no longer have enough energy to reach the plate and the plate current drops to zero and the current to the grid rises as a result of this transfer of current from plate to grid. A graph of grid and total (cathode) current is shown in Fig. 10. It is

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[1]Van der Pol, op. cit., p. 128.

seen that for all values of constant grid voltage the total cathode current drops from its value with zero plate voltage as the plate is made slightly negative, as does the current to the plate. In the same region the grid current rises until the grid and cathode currents become equal at the point where the plate current drops to zero. The curves also show that the plate current drops to zero for negative plate voltages of the order of 0.6 volt. This indicates that the energy due to emission velocity of the most energetic electrons is of the order of 0.6 electron volt.

For somewhat more negative values of plate voltage the electrons that enter the grid-plate space travel almost to the plate before their direction is reversed and they return to the grid. In the region in which they reverse direction these electrons are moving very slowly and they linger long enough to form a negative space charge which results in the same behavior as though the plate were moved closer to the grid. A "virtual" plate is therefore formed which takes a position between the actual plate and the grid.

As the plate voltage is made more negative fewer electrons are admitted to the grid-plate space. The space charge effect is lessened and the grid current tends to hold level, or even to rise slightly. However, as the plate is made more negative it repels the electrons so that few enter the grid-plate space.

Ultimately, as the plate is made highly negative, its action in establishing a repelling field for the electrons reaches well beyond the grid surface. The repelling field eventually overspreads the entire cathode and current to the grid ceases, i.e., a cut-off condition is reached. In this last region the grid current decreases more or less linearly with an increase in the magnitude of the negative plate voltage except for values near the cut-off voltage. Near cut-off the plate potential must influence the field directly between a grid wire and the cathode and a gradual lessening of the rate of decrease of grid current occurs. For this reason the tube exhibits a semi-remote cut-off characteristic.

C. Equipotential Lines

The plots of equipotential lines in a plane-electrode triode, Figs. 5 and 6, can be employed in a qualitative sense to explain the manner in which the grid current varies as a function of negative plate voltage. Care must be used in the interpretation of these equipotential plots because they are for zero space charge conditions.

In the case in which the plate potential is 0 volts, Fig. 5a, electrons which travel in the region approximately midway between the grid wires are accelerated through a potential of about 3.4 volts. As they travel a little past the plane of the grid wires they enter a retarding field. These electrons must lose all the energy they gained from the electric field by the time they reach the plate since the plate and cathode are at the same potential. The electrons that travel this path do, however, reach the plate with some energy because of the emission velocity with which they leave the cathode. Electrons which leave the cathode at points not approximately midway between the grid wires are accelerated in a direction so that they travel to the grids. Those that leave the cathode in the shadow of the grid wires travel in an almost direct line to the grid, while those that leave at points nearer the midpoint between the grids may travel past the grid and then be returned to the grid by the accelerating field. The larger fraction of the total number of electrons travel to the grid. A much smaller fraction go to the plate because the midregion in which the electrons can overcome the attraction of the positive grid is limited in extent.

The equipotential plot in Fig. 5b is of considerable significance because it can be used to formulate a concept of how space charge effects can materially alter the potential distribution. Consider a single electron which leaves the cathode at a point midway between the grid wires. This electron will travel in a straight line upward and midway between the grid wires. As it passes the plane of the grid wires it enters a decelerating field and loses energy until its kinetic energy and velocity are zero at a point slightly above the zero equipotential line. The exact level at which this electron comes to rest will be determined by its emission velocity. The electron is now accelerated back toward the grids and is eventually captured by the grids. It is possible, of course, for the electron to oscillate up and down along the line midway between the grids but such motion is highly unlikely because of slight irregularities which would exist in any actual tube and also as a result of the presence of other electrons.

Now consider the effect of the presence of a large number of electrons in the tube. A certain number of these will be emitted from the cathode so that they will travel past the grid wires and into the grid-plate region. These electrons will follow much the same path as the single electron described above. One important difference should, however, be noted. In the region in which the electrons are being brought to a rest by the electric field these electrons are moving slowly and they are able to establish a space charge determined by the number of electrons in the neighborhood of the region of zero velocity at any one time. This space charge has the effect of providing a repelling field to approaching electrons and causing them to reverse directions at a point closer to the grid wires than in the case of no space charge. The effect, then, is similar to that which would be observed if space charge were negligible and the plate voltage were made somewhat more negative, for the line of zero potential is moved closer to the grid wires by the action of the space charge. Other equipotential lines must also be shifted toward the cathode and the result is to cause an abnormal decrease in grid current.

As the plate potential is made more negative as in Figs. 5c and 6a the line of zero potential approaches the grid plane and parts of this equipotential line may even drop below the grid plane. Under these conditions very few electrons progress into the grid-plate region and the voltage-depressing effect of space charge is very slight. It would be expected that the grid current should decrease linearly with an increase in the size of the negative plate voltage.

The plot of the equipotentials under cut-off conditions is shown in Fig. 6b. This plot does not require modification for space charge effects since these effects are entirely absent when no grid current flows.

An attempt was made to plot the equipotential lines in a plane-electrode tube by use of a model employing graphite-coated paper and brass electrodes. The non-uniformity of the paper made it impossible to obtain results which could be checked by calculations of the type from which Figs. 5 and 6 are plotted. In a strictly qualitative sense the results illustrated the manner in which the equipotential lines changed as the plate voltage was varied. Since mathematical procedures were available for determining the equipotential lines, and neither the mathematical nor the experimental method permitted introduction of the effect of space charge, the experimental method was abandoned in favor of the mathematical method.

D. Oscillations in Inverted Amplifiers

Each of the three inverted amplifiers (Figs. 2, 3 and 4) was investigated analytically to determine if oscillations could arise for any possible load impedance. It was determined that none of these circuits could oscillate unless the equivalent impedance in the input circuit was inductive. The method of analysis did not include the possibility of oscillation as a result of lead inductance. Such oscillations, if they should occur, would be at a very high frequency. Inverted tubes may, however, be used in various oscillator circuits in much the same manner as triodes operated with positive plate and negative grid voltages. In general, the amount of positive feedback voltage provided by the circuit must be higher in the case of the inverted tubes because of the low amplification factor.[1]

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[1] Terman, op. cit., pp. 457, 458.

E. Future Investigations

Several avenues of further investigation of the use and behavior of inverted triodes are suggested by the present study. A more complete explanation is needed for the shapes of the inverse transfer and constant-current curves in the small negative plate voltage region. A study of the effect of space charge with any approximate or exact mathematical procedures that may be applied to the problem would be of help in devising a quantative interpretation of the reasons for the maxima and minima of these curves. As is the case with almost any problem involving space charge effects in systems with relatively complicated geometries, a mathematical approach would be expected to become highly involved. Some simplifying assumptions might be devised which would lead to reasonable estimates of electron paths and velocities. An associated problem is the amount of electron focusing caused by the positively charged grid wires in an inverted triode.

The effect of filament voltage on the characteristics of inverted triodes should be studied in detail. A casual investigation of the effect of filament voltages indicated that the shape of the inverse transfer curves of the 6J5 remained substantially unchanged for filament voltages down to less than one-half the normal value. The value of the grid current dropped along the entire length of the curves but the maxima and minima were still present. A more detailed study of effect of filament voltage might add to an understanding of the underlying reasons for the characteristics of inverted triodes.

The results of the present investigation indicate that many triodes have the same general kind of inverted characteristics and that peculiarities in the inverse transfer curves are common to a wide variety of triodes. Future investigations might encompass a wide variety of triodes for the purpose of determining if all triodes exhibit the same general type of inverted characteristics. Any exceptions that could be found should form a basis for a more complete explanation of the reasons for the characteristics of inverted triodes.

The possibility of building oscillators which utilize inverted triodes has already been suggested. Very little evidence of any considerable amount of work in this area was found in the search of the literature. While it appears that inverted oscillators would generally be simple analogs of conventional oscillators it is possible that oscillators with characteristics different from known oscillators might be developed. In particular, the negative slopes of some of the characteristic curves of inverted triodes might form the basis for a new type of negative resistance oscillator. An oscillator which utilizes the negative slopes of some of the curves would be expected to be somewhat more stable than negative resistance oscillators which employ negative resistances arising from secondary emission effects. At fixed grid and plate supply voltages the characteristic curves of an inverted triode are not subject to changes except those due to aging and these changes would occur slowly. Negative resistances obtained from secondary emission effects tend to be somewhat unstable and are ordinarily subject to rather rapid changes.

The inverted characteristics of multigrid amplifier tubes and the circuits which utilize those tubes offer another field of investigation. No work in this area was indicated in the search of the literature except that of Schneeberger[1] who used an inverted tetrode in a vacuum-tube voltmeter circuit.

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[1]Schneeberger, *op. cit.*, pp. 40-42.

V. SUMMARY

Inverted triodes may be used in several types of amplifier circuits which are analogs of conventional amplifier circuits. These are plate-input grid-output, plate-input cathode-output, and cathode-input grid-output circuits which are analogs, respectively, of plate-loaded, cathode-follower, and grounded-grid amplifiers. In inverted service, the grid of the triode, a 6J5 for example, is operated with a positive supply voltage of from 20 to 50 volts and the plate is operated at a negative potential of from 100 to 250 volts. The roles of the grid and plate, in respect to their functions in conventional amplifiers, are reversed.

The voltage gain of plate-input grid-output and plate-input cathode-output amplifiers is much less than unity and has a maximum magnitude equal approximately to the reciprocal of the normal amplification factor of the tube. The input impedance of these two amplifiers is very high, approaching the limits set by the interelectrode capacitances of the tube, and so the power gain is also very high. The output impedance is approximately equal to the dynamic grid resistance and has a value of about 500 ohms for a 6J5 tube. The large ratio of input to output impedance makes these two inverted amplifiers useful as impedance transformers.

A cathode-input grid-output amplifier has a voltage gain of approximately unity. The input impedance is low and about equal to the load impedance. The output impedance is approximately equal to the sum of the dynamic grid resistance of the tube and the impedance of the signal source.

In inverted service, the grids of voltage amplifier triodes such as the 6J5, 6C5, and 2C22 are capable of passing currents up to 10 ma at a potential of 10 volts with no damage to the grid structure.

The characteristic curves of inverted triodes are regular and linear over a large portion of the operating region. The linearity of the curves and the large biasing voltage on the plate permit the use of signal voltages of the order of 100 to 150 volts (peak) with very low values of harmonic distortion. However, in the regions of the characteristic curves corresponding to small negative plate voltages, of the order of 5 to 15 volts, considerable nonlinearity is found. In these regions both the inverse mutual conductance and inverse amplification factor may be negative. Operation of inverted amplifiers in these regions leads to large amounts of distortion.

The a-c equivalent circuits for inverted triodes are analogs of the equivalent circuits of triodes used in a conventional manner. The roles of the grid and plate are interchanged. The equivalent circuits are useful in the analysis of the behavior of circuits employing inverted triodes.

Inverted triodes may be used in oscillator circuits, but the amount of positive feedback must be much larger than in ordinary oscillators. Very little work has been done in this area.

The use of inverted multigrid tubes has been the subject of very little study. A comprehensive investigation in this field could possibly uncover some novel circuit arrangements with interesting and useful characteristics.

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